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NETWORK DATA ENVELOPMENT ANALYSIS WITH COMMON WEIGHTS: AN APPLICATION TO THE SUSTAINABILITY MEASUREMENT OF OECD COUNTRIES

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Abstract

This work considers a general common-weights network data envelopment analysis (DEA) model which is applicable to most network systems, except those with feedbacks and cycles. The principle of compromise, of the technique for order preference by similarity ideal solution (TOPSIS), is employed to find the common set of weights in the general network DEA model. The proposed method is applied to assess the environmental sustainability performance of the Organization for Economic Co-operation and Development (OECD) countries. Our results show that most countries have a large difference in the rank of efficiencies between the eco-efficiency and production efficiency stages, which reveals the source that causes the low environmental sustainability scores of the whole process.

Keywords: common weights, data envelopment analysis, decision making, sustainability

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1. Introduction

Efficiency measurement has been a subject of tremendous interest in operations research and management science. Data envelopment analysis (DEA) provides a powerful methodology to assess the relative efficiencies of a set of multi-input multi-output decision making units (DMUs), by using a ratio of the weighted outputs to the weighted inputs. Consider a set of comparable DMUs, with each DMU j ($j=1,\cdots,n$) using m inputs X_{ij} ($i=1,\cdots,m$) and generating s outputs Y_{rj} ($r=1,\cdots,s$). The DEA model developed by Charnes et al. (1978), for measuring the technical efficiency of DMU k under the assumption of constant returns to scale in multiplier form, is described as (Eq. 1).

$$\begin{aligned} &\max \sum_{r=1}^{s} u_r Y_{rk} / \sum_{i=1}^{m} v_i X_{ik} \\ &\text{s.t. } \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0,\ j=1,2,\cdots,n, \\ &u_r, v_i \ge \varepsilon,\ r=1,\cdots,s,\ i=1,\cdots,m, \end{aligned} \tag{1}$$

where u_r and v_i are multipliers, and ε is a small non-Archimedean number. This model is referred to as the CCR (Charnes, Cooper and Rhodes) model. Since the advent of DEA in Charnes et al. (1978), various models and applications have been introduced in the literature. Banker et al. (1984) developed the variable returns to scale version of the CCR model, called the BCC model, named after Banker, Charnes, and

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Cooper. Charnes et al. (1985) introduced the additive model which combines both input and output orientations. Tone (2002) proposed the so-called slacks-based measure which is invariant to the units of measurement and is monotone increasing in each input and output slack. For more details on other DEA models and their applications, readers can refer to the work by Cook and Seiford (2009).

In classical DEA models, only the inputs consumed by and the outputs produced from the system are considered, and internal interactions are not taken into account in measuring efficiencies. When a system is composed of several components operating inter-dependently, it has been found that ignoring the operations of the components may produce efficiency measures that are misleading (Castelli et al. 2004). The network DEA is extended from the conventional DEA, which considers the relation and dependencies between internal links, so that the efficiencies are measured more appropriately. This was suggested, for the first time, by Färe and Grosskopf (2000). Then, the multi-stage structures, either with a serial or a parallel or a mixed structure for network DEA models were proposed, based on which models for measuring efficiencies with certain conditions are developed and applications to real world problems are carried out. Overviews of the related network DEA models and applications can be found in Cook and Zhu (2014).

When a huge number of DMUs should be evaluated, decision makers face big data to analyse. Big data first came into prominence in the 1980s and it has now become a "hot" issue in many industries. Recently, some researchers have considered the integration of big data and DEA for evaluating the efficiency of DMUs. Chu et al. (2018) proposed a DEA model and a big data approach for measuring environmental efficiency. Badiezadeh et al. (2018) used a big data approach to assess the sustainability of supply chains using a double frontier network DEA. Li et al. (2017) investigated the efficiency of forestry resources of China based on big data and DEA. Raut et al. (2019) analysed the predictors of sustainable business performance using big data analytics in the context of developing countries. The major challenge of using big data in DEA is the massive number of DMUs and complex interactions between them (Zhu et al., 2018). Common weight models can combat the computational burden of DEA in the big data environment (Mavia et al., 2018).

This work considers developing a common-weights network DEA model which maximizes the system efficiency for all DMUs simultaneously. The proposed model implies DMUs use the same benchmark for calculating efficiencies so that all DMUs can be ranked on a common base. It is shown that the network DEA model with common weights can be reduced into an auxiliary fuzzy bi-objective mathematical programming problem by applying the basic principle of compromise of the technique for order preference by similarity ideal solution (TOPSIS). The fuzzy set theory is then employed to resolve the conflict between two distance objective functions. The case of assessing the environmental sustainability performance of the Organization for Economic Co-operation and Development (OECD) countries, considering the undesirable outputs, is studied. The rest of the paper is organized as follows. The common-weights network DEA model is proposed in Section 2.1. The TOPSIS approach for finding the compromise solution of the proposed network DEA model is presented in Section 2.2. The case of assessing the environmental sustainability performance of OECD countries is studied in Section 3. The paper is concluded in Section 4.

2. Methods

2.1. Common-weights network DEA

Network DEA considers measuring the relative efficiency of a system, taking into account its internal structure. The results are more meaningful and informative than those obtained from the conventional DEA models (Kao, 2014). A variety of network DEA models have been studied to investigate internal interactions of a system and to examine the interrelationship of the components within a system. Since some are similar, they can be classified as being of the same type. For instance, the independent model is concerned with treating the system and component processes independently, while measuring their efficiencies. The ratio-form system efficiency model provides the measurement of the efficiency of system as the ratio of the aggregation of the exogenous outputs to that of the exogenous inputs. In a slacks-based measurement model, the slack variables are adjusted by directional vectors or the range of the corresponding factor. Recently, Kao (2016) proposed a relational network DEA model of general network systems, which is a special type of the ratio-form system efficiency model and is applicable to most network systems, except those with feedbacks and cycles. To describe the general network DEA model, we consider a system composed of q divisions. Any process k, $k = 1,2,\cdots,q$, utilizes exogenous inputs $X_{ij}^{(k)}$ $i \in I^{(k)}$, of DMU j, $j = 1,2,\cdots,n$, supplied from outside and endogenous inputs $\sum_{a=1}^{q} Z_{fj}^{(a,k)}$, $f \in M^{(k)}$, produced by other processes to produce exogenous outputs $Y_{rj}^{(k)}$, $r \in O^{(k)}$, as final outputs of the system and endogenous outputs $\sum_{b=1}^{q} Z_{gj}^{(k,b)}$, $g \in N^{(k)}$, to be utilized by other processes, where $I^{(k)} \subset \{1,2,\cdots, m\}$ and $O^{(k)} \subset \{1,2,\cdots,s\}$ are the index sets for the inputs and outputs of division k, $k = 1,2,\cdots,q$, respectively, and there are m inputs and s outputs in all q divisions. Fig. 1 shows the general structure for a network system.

In Fig. 1, $Z_{fj}^{(a,b)}$ represents the fth intermediate product produced by division a for division b to use. $\sum_{a=1}^{q} Z_{fj}^{(a,k)}$ is the total amount of the fth intermediate product, $f \in M^{(k)}$, produced by other divisions for division k to use, and $\sum_{b=1}^{q} Z_{gj}^{(k,b)}$ is then the total amount of the gth intermediate product, $g \in N^{(k)}$, produced by division k for other divisions to use, where $\mathrm{M}^{(k)} \subset \{1,2,\cdots,h\}$ are the index sets for the intermediate products to be consumed and produced by division k, respectively, and there are h intermediate products in all q divisions.

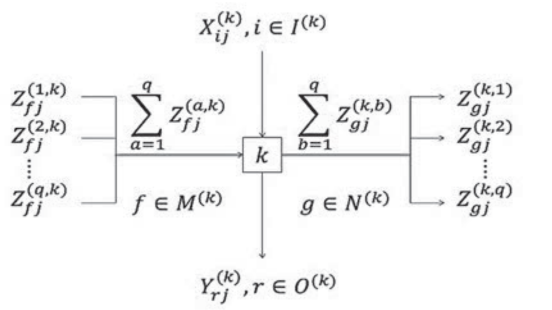

Fig. 1. A general network system (Kao, 2016)

The relational network DEA model, where the system efficiency is decomposed into a weighted average of the division efficiencies adjusted by a factor, can be formulated as (Eq. 2).

$$
\begin{aligned}
&\max \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{r0}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{i0}^{(k)} \\
&\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j = 1,2,\cdots,n, \\
&[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0, \\
&k = 1,2,\cdots,q,\ j = 1,2,\cdots,n, \\
&u_r, v_i, w_g \ge \varepsilon,\ \forall\, r,i,g. \qquad (2)
\end{aligned}
$$

Similar to the conventional DEA methods, the network DEA model (Eq. 2) selects weights attached to inputs and outputs for each individual DMU. Using different sets of weights to compare the DMUs may not be rational or acceptable to most practitioners. To eliminate the inconsistency caused by using different facets to calculate efficiency, the common-weights network DEA model, which maximizes the system efficiencies for all DMUs simultaneously, is considered as (Eq. 3).

$$
\begin{aligned}
&\max\{\sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{r1}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{i1}^{(k)}, \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{r2}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{i2}^{(k)}, \cdots, \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rn}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{in}^{(k)}\} \\
&\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j = 1,2,\cdots,n, \\
&[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0, \\
&k = 1,2,\cdots,q,\ j = 1,2,\cdots,n, \\
&u_r, v_i, w_g \ge \varepsilon,\ \forall\, r,i,g. \qquad (3)
\end{aligned}
$$

It should be noted that (Eq. 3) is a multiple objective optimization problem. Most multiple objective decision-making (MODM) problems cannot be optimized simultaneously due to the inherent incommensurability and conflict between these objectives. Obtaining a compromise solution for multiple objective functions is usually a major topic for solving MODM problems. Goal programming and global criterion methods are some of the most popular approaches in the literature (Lai and Hwang, 1994). These methods consider only one criterion based on the shortest distance from the given goal as the positive ideal solution. However, in practice, such a single criterion may not be sufficient for a policy maker. TOPSIS was first developed by Hwang and Yoon (1981), to solve a multiple attribute decision making problem. The principle of compromise of TOPSIS is based on the premise that the chosen alternative should have "the shortest distance from the positive ideal solution (PIS)" and "the farthest distance from the negative ideal solution (NIS)." This principle of compromise was later suggested by Hwang et al. (1993) for solving MODM problems. In this work, TOPSIS is employed for finding the common set of weights in the general network DEA model. At optimality, the system and division efficiencies are calculated as (Eqs. 4-5), respectively (Kao, 2016).

$$
E_0 = \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{r0}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{i0}^{(k)}. \qquad (4)
$$

$$
E_0^{(k)} = \frac{\sum_{r=1}^{s} u_r Y_{r0}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{g0}^{(k,b)})}{\sum_{i=1}^{m} v_i X_{i0}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{f0}^{(a,k)})},\ k = 1,2,\cdots,q. \qquad (5)
$$

2.2. A compromise solution approach

In this section, the TOPSIS approach for finding the compromise solution of the common-weights network DEA model (Eq. 3) is introduced. The principle of compromise of TOPSIS considers the chosen solution should have "the shortest distance from the PIS" and "the farthest distance from the NIS." To formulate this principle, the reference points of PIS and NIS of the problem (Eq. 3) are defined as (Eqs. 6-7), respectively.

$$
\begin{aligned}
&E_j^{*} = \max \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \\
&\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j = 1,\ 2,\cdots,n, \\
&[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0, \\
&k = 1,2,\cdots,q,\ j = 1,2,\cdots,n, \\
&u_r, v_i, w_g \ge \varepsilon,\ \forall\, r,i,g. \qquad (6)
\end{aligned}
$$

$$E_j^- = \min \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} / \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)}$$
$$\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j=1,\ 2,\cdots,n,$$
$$[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0,$$
$$k=1,2,\cdots,q,\ j=1,2,\cdots,n,$$
$$u_r, v_i, w_g \ge \varepsilon,\ \forall\ r,i,g. \tag{7}$$

The PIS of model (Eq. 3), say E^*, is defined as the solution vector of Eq. (6), i.e., $E^* = (E_1^*, E_2^*, \cdots, E_n^*)^{\mathrm{T}} \in R^n$. And the NIS of model (Eq. 3), say E^-, is defined as the solution vector of equation (Eq. 7), i.e., $E^- = (E_1^-, E_2^-, \cdots, E_n^-)^{\mathrm{T}} \in R^n$. With PIS and NIS of the model (Eq. 3), the distance from PIS to all objectives, say d_p^{PIS}, and the distance from NIS to all objectives, say d_p^{NIS}, are defined by employing the Minkowski's L_p-metric and described as (Eqs. 8-9), respectively.

$$d_p^{PIS} = \{\sum_{j=1}^{n} \lambda_j^p [\frac{E_j^* - E_j}{E_j^* - E_j^-}]^p\}^{1/p} \tag{8}$$

$$d_p^{NIS} = \{\sum_{j=1}^{n} \lambda_j^p [\frac{E_j - E_j^-}{E_j^* - E_j^-}]^p\}^{1/p} \tag{9}$$

where: $\lambda_j \in [0,1],\ j=1,2,\cdots,n,$ is the degree of importance of the jth objective, and p is the parameter of distance functions, $p=1,2,\cdots,\infty.$

The TOPSIS approach suggests to approximately minimize the distances from PIS to all objectives and approximately maximize the distance from NIS to all objectives, to find a compromise solution of the common-weights general network DEA model (Eq. 3). The fuzzy bi-objective optimization problem (Eq. 10) is then considered.

$$\widetilde{\min}\ d_p^{PIS}$$
$$\widetilde{\max}\ d_p^{NIS}$$
$$\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j=1,\ 2,\cdots,n,$$
$$[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0,$$
$$k=1,2,\cdots,q,\ j=1,2,\cdots,n,$$
$$u_r, v_i, w_g \ge \varepsilon,\ \forall\ r,i,g, \tag{10}$$

where: $p=1,2,\cdots,\infty.$ Each fuzzy objective of the problem (Eq. 10) can be represented by a fuzzy set with the corresponding membership function defined as (Eqs. 11-12).

$$\mu_{PIS} = \begin{cases} 1, & \text{if } d_p^{PIS} \le (d_p^{PIS})^* \\ \dfrac{(d_p^{PIS})' - d_p^{PIS}}{(d_p^{PIS})' - (d_p^{PIS})^*}, & \text{if } (d_p^{PIS})^* \le d_p^{PIS} \le (d_p^{PIS})' \\ 0, & \text{if } d_p^{PIS} > (d_p^{PIS})' \end{cases} \tag{11}$$

$$\mu_{NIS} = \begin{cases} 1, & \text{if } d_p^{NIS} > (d_p^{NIS})^* \\ \dfrac{d_p^{NIS} - (d_p^{NIS})'}{(d_p^{NIS})^* - (d_p^{NIS})'}, & \text{if } (d_p^{NIS})' \le d_p^{NIS} \le (d_p^{NIS})^* \\ 0, & \text{if } d_p^{NIS} < (d_p^{NIS})' \end{cases} \tag{12}$$

where: $(d_p^{PIS})^* = \min\ d_p^{PIS}$

$$\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j=1,\ 2,\cdots,n,$$
$$[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0,$$
$$k=1,2,\cdots,q,\ j=1,2,\cdots,n,$$
$$u_r, v_i, w_g \ge \varepsilon,\ \forall\ r,i,g, \tag{13}$$

with the solution denoted by $(u,w,v)^{PIS}$,

$(d_p^{NIS})^* = \max\ d_p^{NIS}$

$$\text{s.t. } \sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j=1,\ 2,\cdots,n,$$
$$[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0,$$
$$k=1,2,\cdots,q,\ j=1,2,\cdots,n,$$
$$u_r, v_i, w_g \ge \varepsilon,\ \forall\ r,i,g, \tag{14}$$

with the solution denoted by $(u,w,v)^{NIS}$, $(d_p^{PIS})'$ is defined as (Eq. 15),

$$(d_p^{PIS})' = d_p^{PIS}((u,w,v)^{NIS}), \tag{15}$$

and $(d_p^{NIS})'$ is defined as (Eq. 16),

$$(d_p^{NIS})' = d_p^{NIS}((u,w,v)^{PIS}). \tag{16}$$

According to Bellman and Zadeh (1970), a solution of the fuzzy bi-objective optimization problem (Eq. 10) can be obtained by solving the problem (Eq. 17).

$$\max \alpha$$
$$\text{s.t. } \mu_{PIS} \ge \alpha,$$
$$\mu_{NIS} \ge \alpha,$$
$$\sum_{k=1}^{q}\sum_{r=1}^{s} u_r Y_{rj}^{(k)} - \sum_{k=1}^{q}\sum_{i=1}^{m} v_r X_{ij}^{(k)} \le 0,\ j=1,\ 2,\cdots,n,$$
$$[\sum_{r=1}^{s} u_r Y_{rj}^{(k)} + \sum_{g=1}^{h} w_g (\sum_{b=1}^{q} Z_{gj}^{(k,b)})] - [\sum_{i=1}^{m} v_i X_{ij}^{(k)} + \sum_{f=1}^{h} w_f (\sum_{a=1}^{q} Z_{fj}^{(a,k)})] \le 0,$$
$$k=1,2,\cdots,q,\ j=1,2,\cdots,n,$$
$$u_r, v_i, w_g \ge \varepsilon,\ \forall\ r,i,g, \tag{17}$$

where: α is the degree of satisfactory level for both fuzzy objectives of (Eq. 10).

3. Case studies

In this work, the proposed method is applied to assess the environmental sustainability performance of OECD countries and to rank the results. A large number of nations adopted sustainable development goals in the United Nations' Earth Summit in September 2015. Sustainability is multidimensional and encompasses economic, social, and environmental aspects. An important instrument of sustainable development is eco-efficiency. Evaluation models and applications on the joint investigation of economic growth and environmental impact have been developed recently. For instance, Sun et al. (2019) proposed a game meta-frontier DEA model for evaluating the circular economic system in China. Gardas et al. (2018) developed a Delphi-DEMATEL approach for modelling the challenges to sustainability in the textile and apparel sector. Tian et al. (2020) measured the regional transport sustainability, using super-efficiency SBM-DEA with weighting preference. According to Halkos et al. (2016), the sustainability efficiency measurement can be decomposed into two stages: the production efficiency process and the eco-efficiency process, which encompasses both economic and ecological aspects. In the production efficiency stage, the two inputs, labour force (X_1) and capital stock (X_2), are utilized to produce a desirable output. The real gross domestic product (GDP) measures, based on multiple purchasing power parity benchmarks, is an appropriate measure of output across countries in the first stage. In addition, it is the intermediate variable (Z) in our model, and it is used as an input in the second stage. In the second stage, the eco-efficiency process, environmental pressures are incorporated as undesirable outputs. It is well known that greenhouse gases absorb and re-emit thermal radiation, which causes global warming. In this study, the most important greenhouse gases are employed as a measure for environmental pressures, which are carbon dioxide CO_2 (Y_1), methane CH_4 (Y_2), and nitrous oxide N_2O (Y_3), which are all measured in gigagrams of the CO_2 equivalent. Fig. 2 shows the two-stage sustainability efficiency measurement system. This study uses the data of the sustainability efficiency assessment of OECD countries, obtained from the 2017 World Bank report (W.B. Report, 2017). Table 1 lists the inputs, the intermediate products, and the outputs of the 34 OECD countries.

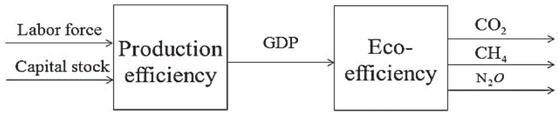

Fig. 2. Network structure of the study

Table 1. Data of the study

DMU	*Labor force*	*Captial stock*	*GDP*	CO_2	CH_4	N_2O
1.Australia	12026236	421910703237	1543411012579.91	388126.3	125588.2	54247.48
2. Austria	4352343	92731920022	409425234155.263	62272.99	8006.814	3789.945
3. Belgium	4912346	112646178564	497884216568.867	95107.31	9243.349	8502.954
4. Canada	19264216	447922574398	1824288757447.57	517457.7	106846.8	33413.53
5. Chile	8379169	66448645672	267122320056.702	80974.69	18380.5	8949.187
6. Czech Rep	5269165	53732243647	207376427020.815	101029.5	11901.93	7315.461
7. Denmark	2911871	61431075146	327148899962.146	36427.98	7602.608	5340.426
8. Estonia	684718	6588082942	23043864510.0543	17623.6	2234.878	951.011
9. Finland	2699346	57317701082	256706466091.089	49134.13	8551.563	5547.307
10. France	30091903	602736549739	2683825225092.63	333227.6	81178.5	36865.68
11. Germany	41807485	712772145361	3543983909148.01	739861.3	55720.82	43410.64
12. Greece	5040033	31016564129	245670666639.047	80043.28	8254.87	4639.345
13. Hungary	4368797	24644888107	127856647107.82	44583.39	7134.917	4356.41
14. Iceland	189265	2274545279	14292008745.4017	1800.497	358.7683	368.9622
15. Ireland	2183707	2274545279	225571853194.348	35591.9	14330.03	7302.29
16. Israel	3641401	53607108586	257296579579.346	75529.2	3415.643	1750.534
17. Italy	24962311	380532101847	2072823157059.76	369468.6	35238.18	20083.84
18. Japan	65650470	1390713957202	6203213121334.12	1230168	38956.54	24911.49
19. Korea Rep	26054417	361577858920	1222807284485.31	583966.1	32624.7	14979.34
20. Luxembourg	255227	11428008638	56677961787.0717	10663.64	1169.316	473.0239
21. Mexico	53380542	274334203728	1201089987015.45	496324.8	116704.6	43436.27
22. Netherlands	8962768	156660673785	828946812396.788	170310.1	19025.82	8924.044
23. New Zealand	2362576	36225060437	176192886551.397	34150.77	28657.66	11879.94
24. Norway	2671604	114364933390	510229136226.902	49889.54	16408.94	3305.092
25. Poland	18229612	99002763279	500284003684.372	299931.3	65071.2	26721.64
26. Portugal	5388411	34269810199	216368178659.447	46013.52	12976.02	4010.799
27. Slovak Rep	2710517	19845861474	93413992955.8972	32764.65	4074.574	2778.4
28. Slovenia	1016608	8909139413	46352802765.5763	14781.68	2821.745	1165.984
29. Spain	23604153	264474742727	1336018949805.58	264779.4	37208.11	20873.14
30. Sweden	5033981	123125911513	543880647757.404	47047.61	10304.22	5221.58
31. Switzerland	4597385	159179127596	668043614122.87	37773.77	4900.251	2385.447
32. Turkey	26756052	238770483853	873982182628.062	329560.6	78852.94	35611.71
33. UK	32543044	415166653701	2662085168498.93	468572.9	58980.03	25334.92
34. USA	158429022	3064346200000	16155255000000	5119436	499809.3	288878

To assess the environmental sustainability performance of OECD countries, the following common-weights general network DEA model (Eq. 18) is considered.

$$\max\left\{\frac{u_1Y_{11}+u_2Y_{21}+u_3Y_{31}}{v_1X_{11}+v_2X_{21}},\frac{u_1Y_{12}+u_2Y_{22}+u_3Y_{32}}{v_1X_{12}+v_2X_{22}},\cdots,\frac{u_1Y_{1,34}+u_2Y_{2,34}+u_3Y_{3,34}}{v_1X_{1,34}+v_2X_{2,34}}\right\}$$

$$\begin{aligned}\text{s.t. } & w\ Z_j - v_1X_{1j} - v_2X_{2j} \le 0,\ j=1,2,\cdots,34,\\ & u_1Y_{1j}+u_2Y_{2j}+u_3Y_{3j} - w\ Z_j \le 0,\ j=1,2,\cdots,34,\\ & u_1,u_2,u_3,v_1,v_2,w \ge \varepsilon.\end{aligned} \tag{18}$$

The results of PIS, E_j^*, and NIS, E_j^-, $j=1,2,\cdots,34$, calculated by solving (Eqs. 8-9) are shown in the Appendix.

With the data in the Appendix, we then measure the distance from the PIS and NUS to all objectives. Taken $p=2, \lambda_1=\lambda_2=\cdots=\lambda_{34}=\frac{1}{34}$, we have $(d_2^{PIS})^*=0.1607, (d_2^{NIS})^*=0.0705$, $(d_2^{PIS})'=0.1617, (d_2^{NIS})'=0.0572.$ Then the membership functions of the two distance objective functions can be described as (Eqs. 19-20).

$$\mu_{PIS}=\begin{cases}1, & \text{if } d_2^{PIS}\le 0.1607\\ \dfrac{0.1617-d_2^{PIS}}{0.001}, & \text{if } 0.1607\le d_2^{PIS}\le 0.1617\\ 0, & \text{if } d_2^{PIS}>0.1617\end{cases} \tag{19}$$

$$\mu_{NIS}=\begin{cases}1, & \text{if } d_2^{NIS}>0.0705\\ \dfrac{d_2^{NIS}-0.0138}{0.0133}, & \text{if } 0.0572\le d_2^{NIS}\le 0.0705\\ 0, & \text{if } d_2^{NIS}<0.0572\end{cases} \tag{20}$$

A compromise solution of the common-weights general network DEA model (Eq. 18) can then be obtained by solving the problem (Eq. 21).

$$\begin{aligned}&\max \alpha\\ \text{s.t. } & d_2^{PIS}+0.001\alpha-0.1617\le 0,\\ & -d_2^{NIS}+0.0133\alpha+0.0572\le 0,\\ & w\ Z_j - v_1X_{1j}-v_2X_{2j}\le 0,\ j=1,2,\cdots,34,\\ & u_1Y_{1j}+u_2Y_{2j}+u_3Y_{3j}-w\ Z_j\le 0,\ j=1,2,\cdots,34,\\ & u_1,u_2,u_3,v_1,v_2,w\ge\varepsilon,\ \ \alpha\in[0,1].\end{aligned} \tag{21}$$

The right half of Table 2, under the heading "Common-weights network DEA model", shows the overall efficiencies E_j, the efficiency of the first sub-process E_j^1, and the efficiency of the second sub-process E_j^2, calculated from the proposed compromise solution approach. The numbers in parentheses are the rankings of the corresponding DMUs. Since the overall efficiency E_j is the product of the efficiencies of the first stage E_j^1 and the second stage E_j^2, every E_j is no greater than its corresponding E_j^1 and E_j^2. It should be noted that none of the 34 OECD countries perform efficiently in both stages. This is seen in the non-efficient overall scores of all the OECD countries, as shown in the fifth column in Table 2. Ireland and Luxembourg are efficient in Stage 1, and Iceland is the only efficient DMU in Stage 2, as shown in the last two columns of Table 2. The last row of Table 2 shows the averages of these three measures. The efficiency scores in Stage 2 are significantly lower than those in Stage 1, which reveals that the OECD countries' high production efficiency does not ensure a high eco-efficiency performance.

To investigate the rank of efficiencies E_j, E_j^1 and E_j^2, which are the numbers in parentheses, we see that most countries have a large difference in the rank of efficiencies between the eco-efficiency and production efficiency stages. It reveals the source that causes the low environmental sustainability scores of the whole process. For instance, the USA performs unsatisfactorily in Stage 2 (as compared to Stage 1) and Iceland performs unsatisfactorily in Stage 1 (as compared to Stage 2). However, some countries have similar ranks in E_j, E_j^1 and E_j^2, for example, Finland and Luxembourg. This implies that the performance of the whole process is evenly attributed to the performance of the two sub-processes in these countries.

By applying (Eqs. 4-5), the overall efficiencies E_j, the efficiency of the first sub-process E_j^1, and the efficiency of the second sub-process E_j^2 of the OECD countries are calculated. These scores are the highest values that the DMUs can attain. They are regarded as the positive ideal solution in the proposed method. The results are shown in the left half of Table 2, under the heading "Model (Eq. 2)." The highest environmental sustainability performance occurs in Norway. Ireland is production efficient, and Iceland is eco-efficient. To compare the results obtained from Model (Eq. 2) and the common-weights general network DEA model, it is more appropriate to compare the ranks, rather than the efficiency scores.

The rankings of the overall efficiencies of the two models are quite similar. The correlation of rank position between the two models is high, with a calculated coefficient of 0.7488, indicating that the results of the proposed method are reasonable. In this analysis, we used the most widely used Pearson correlation coefficient, which is the covariance of the two variables divided by the product of their standard deviations.

Figs. 4-6 show the associated rankings of the overall efficiencies, the production efficiencies, and the eco-efficiencies of OECD countries calculated from the two approaches, respectively.

Table 2. Efficiency scores and the associated rankings (in parentheses) calculated from different models

DMU	***Model (Eq. 2)***			***Common-weight network DEA***		
J	E_j	E_j^1	E_j^2	E_j	E_j^1	E_j^2
1. Australia	0.2112(15)	0.6539(5)	0.3230(25)	0.0641(21)	0.6556(7)	0.0978(29)
2. Austria	0.2267(12)	0.5789(12)	0.3916(20)	0.0823(14)	0.5941(14)	0.1385(18)
3. Belgium	0.2165(13)	0.6199(9)	0.3492(22)	0.0802(16)	0.6228(10)	0.1287(19)
4. Canada	0.1531(20)	0.4276(19)	0.3580(21)	0.0542(25)	0.5788(16)	0.0937(30)
5. Chile	0.1209(28)	0.2614(29)	0.4627(17)	0.0389(31)	0.2613(31)	0.1489(14)
6. Czech Rep	0.2429(11)	0.2540(30)	0.9562(2)	0.0564(23)	0.3071(27)	0.1837(9)
7. Denmark	0.4845(3)	0.7014(4)	0.6908(7)	0.1041(9)	0.7121(5)	0.1462(15)
8. Estonia	0.1930(17)	0.2649(28)	0.7286(5)	0.1701(3)	0.2658(29)	0.6398(2)
9. Finland	0.3849(4)	0.6018(10)	0.6396(8)	0.0936(10)	0.6014(12)	0.1557(31)
10. France	0.1216(27)	0.4272(20)	0.2846(27)	0.0491(27)	0.5759(18)	0.0853(31)
11. Germany	0.1085(31)	0.5773(13)	0.1879(33)	0.0485(28)	0.5776(17)	0.0840(32)
12. Greece	0.1886(18)	0.3111(26)	0.6062(11)	0.0700(19)	0.4160(23)	0.1682(11)
13. Hungary	0.1281(23)	0.1958(32)	0.6218(9)	0.0662(20)	0.2528(32)	0.2618(6)
14. Iceland	0.5673(2)	0.5673(15)	1.0000(1)	0.5673(1)	0.5673(19)	1.0000(1)
15. Ireland	0.2667(8)	1.0000(1)	0.2667(28)	0.1634(4)	1.0000(1)	0.1634(12)
16. Israel	0.1219(24)	0.5030(18)	0.2423(31)	0.0732(18)	0.5031(21)	0.1455(16)
17. Italy	0.1514(21)	0.5847(11)	0.2589(30)	0.1504(5)	0.5853(15)	0.2569(7)
18. Japan	0.1082(32)	0.5771(14)	0.1875(34)	0.0492(26)	0.5980(13)	0.0823(33)
19. Korea Rep.	0.0967(33)	0.3342(25)	0.2892(26)	0.0401(30)	0.3397(26)	0.1179(26)
20. Luxembourg	0.3403(6)	0.9880(2)	0.3444(23)	0.3372(2)	1.0000(1)	0.3372(4)
21. Mexico	0.1217(26)	0.1977(33)	0.6154(10)	0.0231(34)	0.1966(34)	0.1176(27)
22. Netherlands	0.2115(14)	0.6229(8)	0.3395(24)	0.0745(17)	0.6352(9)	0.1192(24)
23. New Zealand	0.2430(10)	0.5233(16)	0.4643(16)	0.1144(7)	0.5248(20)	0.2180(8)
24. Norway	0.6059(1)	0.8637(3)	0.7056(6)	0.1111(8)	0.8812(3)	0.1261(21)
25. Poland	0.0771(34)	0.1022(34)	0.7547(4)	0.0306(33)	0.2382(33)	0.1286(20)
26. Portugal	0.2603(9)	0.3365(24)	0.7736(3)	0.0590(22)	0.341(25)	0.1730(10)
27. Slovak Rep	0.1218(25)	0.2863(27)	0.4253(18)	0.0865(13)	0.2864(28)	0.3019(5)
28. Slovenia	0.1533(19)	0.3669(23)	0.4179(19)	0.1388(6)	0.3669(24)	0.3783(3)
29. Spain	0.1136(30)	0.4308(21)	0.2636(29)	0.0461(29)	0.4324(22)	0.1067(28)
30. Sweden	0.3431(5)	0.6429(6)	0.5344(12)	0.0811(15)	0.6474(8)	0.1253(22)
31. Switzerland	0.2724(7)	0.5197(17)	0.5241(13)	0.0914(11)	0.7471(4)	0.1224(23)
32. Turkey	0.1190(29)	0.2528(31)	0.4709(15)	0.0310(32)	0.2619(30)	0.1184(25)
33. UK	0.1967(16)	0.3993(22)	0.4825(14)	0.0880(12)	0.6054(11)	0.1454(17)
34. USA	0.1298(22)	0.6300(7)	0.1918(32)	0.0544(24)	0.6665(6)	0.0816(34)
Average	0.2177	0.4884	0.4751	0.0100	0.5249	0.1970

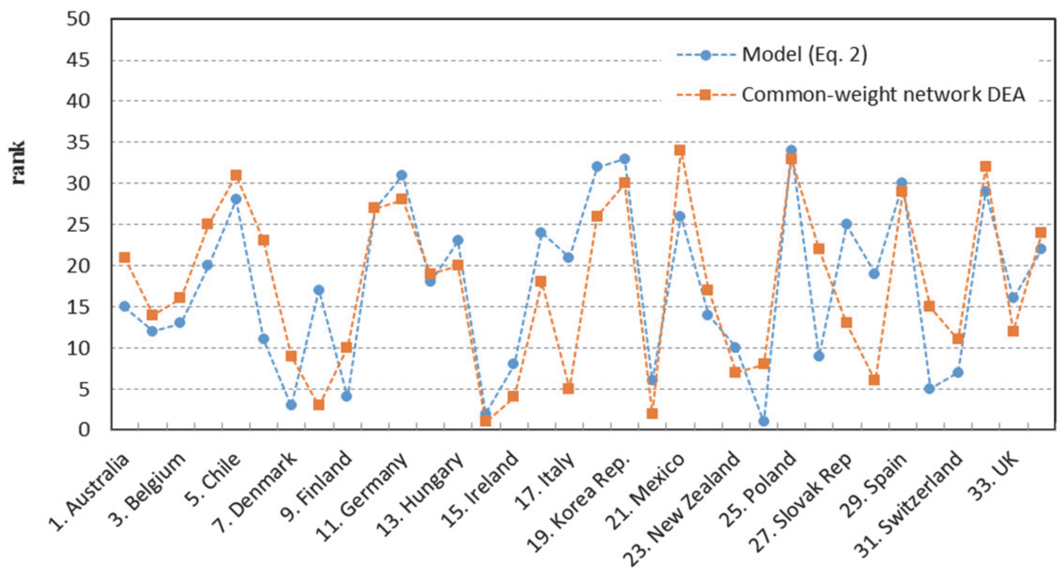

Fig. 4. The ranking of overall efficiencies of OECD countries

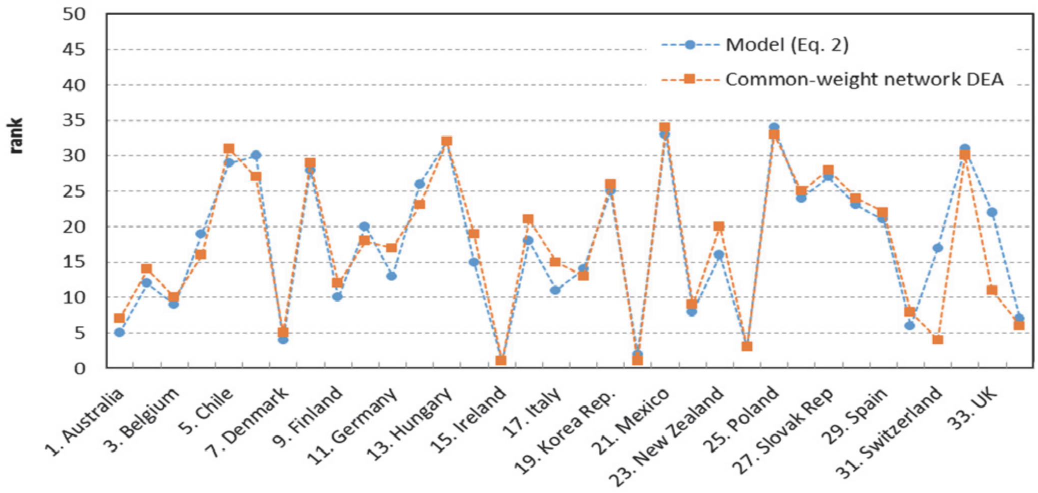

Fig. 5. The ranking of production efficiencies of OECD countries

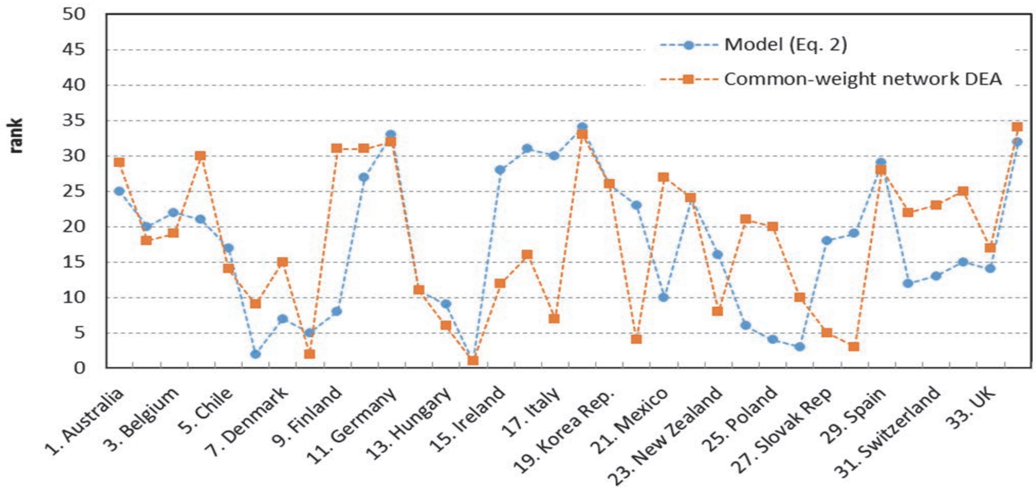

Fig. 6. The ranking of eco-efficiencies of OECD countries

4. Conclusions

This work provides a common base for measuring the efficiencies of a group of DMUs in a general network system. Applying the basic principle of compromise of TOPSIS, the *n*-objective common-weights network DEA model is reduced to an auxiliary fuzzy bi-objective optimization problem. The fuzzy set theory is then employed to resolve the conflict between two distance objective functions and to find the compromise solution of the common-weights network DEA model. Our approach effectively overcomes the shortcomings highlighted for the conventional DEA models, by providing a common base for measuring the individual and overall efficiencies of a general network system appropriately. The proposed model is applied to assess the environmental sustainability performance of OECD countries and to rank the results. Our results show that most countries have a large difference in the rank of efficiencies between the eco-efficiency and production efficiency stages, which reveals the source that causes the low environmental sustainability scores of the whole process. This is valuable information for policy makers, who can assess which individual stage they should aim to improve, for enhancing the overall environmental sustainability performance of a country.

The DEA window analysis on the application in Section 3, can be performed to explore the effect of input data change in the proposed model for future studies. The analysis including social and culture dimensions on the application, can also be a possible interesting research topic for studying the sustainability performance of OECD countries. Furthermore, the relational model used in this paper, for measuring efficiencies of general network systems, is a type of radial model. There are also non-radial models, for example, the slacks-based measurement. Developing common-weights network DEA models, based on other types of models, could also be a direction for future research.

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Appendix

Table 3. The positive ideal solution E_j^* and the negative ideal solution E_j^-

DMU	E_j^*	E_j^-	***DMU***	E_j^*	E_j^-
1	0.2112	$3.2904*10^{-4}$	18	0.1082	$1.4958*10^{-5}$
2	0.2267	$1.6314*10^{-4}$	19	0.0967	$3.5662*10^{-4}$
3	0.2165	$1.0612*10^{-4}$	20	0.3403	$5.2335*10^{-4}$
4	0.1531	$1.7224*10^{-4}$	21	0.1217	$1.2956*10^{-5}$
5	0.1209	$1.3824*10^{-4}$	22	0.2115	$3.2809*10^{-5}$
6	0.2429	$1.3718*10^{-4}$	23	0.2430	$1.2468*10^{-4}$
7	0.4845	$1.5151*10^{-4}$	24	0.6095	$8.4077*10^{-4}$
8	0.1930	$2.6539*10^{-4}$	25	0.0771	$8.2966*10^{-6}$
9	0.3349	$1.9811*10^{-4}$	26	0.2603	$8.9131*10^{-4}$
10	0.1216	$3.6624*10^{-6}$	27	0.1218	$5.0258*10^{-4}$
11	0.1085	$3.6067*10^{-5}$	28	0.1533	$5.5826*10^{-4}$
12	0.1886	$1.1452*10^{-4}$	29	0.1136	$6.2386*10^{-6}$
13	0.1281	$2.1704*10^{-4}$	30	0.3431	$5.9674*10^{-4}$
14	0.5673	$9.3655*10^{-4}$	31	0.2724	$5.3038*10^{-6}$
15	0.2667	$1.2314*10^{-4}$	32	0.1190	$2.3279*10^{-5}$
16	0.1219	$1.8335*10^{-5}$	33	0.1967	$1.2343*10^{-4}$
17	0.1514	$3.4278*10^{-6}$	34	0.1298	$4.9398*10^{-6}$