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OPTIMAL DESIGN OF CASCADE SPILLWAY USING META-HEURISTIC ALGORITHMS: COMPARISON OF FOUR DIFFERENT ALGORITHMS

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Abstract

In the present research, four meta-heuristic algorithms named Genetic Algorithm (GA), Gravitational Search Algorithm (GSA), Particle Swarm Optimization (PSO), and Artificial Bee Colony (ABC) have been used for optimal design of cascade spillway to minimize the construction cost. Usually, the traditional design methods such as Vittal and Porey (VP) method or experimental modeling are used to solve this problem leading to infeasible or near optimal solution. The main novelty of this paper is to use effective methods to solve this complex highly constrained problem. Therefore, due to unique features of meta-heuristic algorithms, these algorithms are used here to minimize the construction cost of cascade spillway as the energy dissipater structure. As the case study, cascade spillway of Tehri dam in India had been chosen. The algorithms results have been compared together and with the VP results. Comparison of the results show the effectiveness and affectivity of these algorithms to solve this optimization problem. In other words, when three-stepped spillway are considered, the results are improved with 16.16%, 16.4%, 17.73% and 17.63% respectively using GA, GSA, PSO and ABC algorithms and in the same manner, for four-stepped spillway, the results are improved 14.5%, 16.1%, 16.45% and 16.05% respectively using GA, GSA, PSO and ABC algorithms in comparison with the VP results.

Keywords: cascade spillway, energy dissipater structures, meta-heuristic algorithm, optimal design, Tehri dam

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1. Introduction

Generally, dams are constructed for different purposes such as water storage, hydro-power generation, fishing, recreational. Dam's structure consists of different parts in which one of them is an energy dissipater structure. Generally, there are significant differences between upstream and downstream water elevations, and therefore lots of potential energy are stored in water. However, a great deal amount of this energy should be reduced to prevent collapsing downstream structures and other problems. In general, energy dissipation can perform with some structures such as spillways and stilling basins (Bakhtyar et al., 2007). Design and modeling of dams consist of several issues including body, reservoir, spillway, power planet, tunnels. Nowadays, numerous researches have been done in this field using different methods such as experimental modeling or structural and hydraulic software modeling and design (Buffi et al., 2017; Brown, 2017; Castillo and Carillo, 2017; Chen et al., 2018; Wang and Jia, 2017; Wang et al., 2019). Here, optimal design of stepped spillway is considered and therefore, reviewing the other research field for dam modeling is not worthful and not presented here to avoid lengthy paper.

The spillway is a hydraulic structure that transfers surplus water from upstream to downstream. Spillways have different types such as siphon

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spillway, lateral spillway, Ogee spillway, cascade spillway and so on. For large dams, a single spillway is not useful to reduce energy. Because of the high elevation and fast velocity of flow, Froud Number at the toe of spillway chute will be magnified. To solve this problem, a hybrid system compounded of some stepped spillway and succeeding stilling basin is an efficient solution so it can use cascade spillways (Vittal and Porey, 1987). It should be noted that most of the cascade spillways are design as a prismatic section, so they are more economical and have better hydraulic performances that the other spillways and therefore, it is considered in this research (Bozorg Haddad et al., 2005).

Generally, literature review of different spillways such as cascade spillway are divided into two main categories, the first category discusses hydraulic conditions of spillways and tries to maximize the hydraulic performance in order to improve the hydraulic characteristics of spillways regardless of construction costs. Many research including experimental and software modeling and hydraulic simulation have been done in this category in which some of them are presented below.

Barani et al. (2005) made a wooden model to consider energy dissipation in cascade spillways. This model has different forms such as simple stepped, step with inverse incline, step with end sill. The results indicated that spillways with inverse incline and end sill have better energy dissipation than simple spillway. The correlation between the steps number and energy reduction was investigated by Chinnarasri and Wongwises (2006) and Tabari and Tavakoli (2016) so what when the steps number decreases the energy dissipation increases. As another results, while the step heights increase and the flow discharge decreases the energy reduction improves. In general, for obtaining maximum hydraulic performance, the cavitations should be removed. Frizell et al. (2012) indicated that cascade spillway is more resistant than other spillway and when the slope increases the probability of cavitations occurrence reduces. Due to this fact, dam designers are encouraged to design and use cascade spillway. Shahheydari et al. (2014) have considered 112 numerical models for spillway with Flow-3D software (96 models for cascade spillways and 16 models for simple spillway). The results showed that energy dissipation is related to flow coefficient so that while flow coefficient increases, the rate of energy dissipation decreases. In addition, Roushangar et al. (2014) used the Artificial Neural Networks (ANNs) and Genetic Expression Programming (GEP) to predict energy dissipation for the stepped spillway in two different regimes using experimental dataset. As finding the dimension and final shape of the stepped spillway was the hard task, machine learning was a suitable approach to predict energy decrease. Furthermore, the energy reduction over the stepped spillway was investigated using horizontal, inclined, and horizontal curved steps (Mero and Mitchel, 2017). As the result, the energy dissipation over the inclined and horizontal curved steps had the energy reduction two times more than the horizontal steps. In addition, three forms of breakers such as Over-flow; through-flow and under-flow breakers were regarded to evaluate the effects on the stepped spillway (Aal et al., 2017). The results highlighted that the energy dissipation over the stepped spillway with breakers had more energy reduction in comparison with the classical stepped spillway. Finally, Afshoon et al. (2019) used Flow-3D software to show the effect of the roughness of the steps of spillway on the flow energy.

The second category is focused on the optimal design of spillway. Although, many research have been done in the field of hydraulic performance of spillways (first category) however, this research is focused on the second research category for spillways means minimize the construction cost of this structure. To design the cascade spillway, this could be modeled as an optimization problem and could be solved using optimization methods. In general, these methods can be categorized as Linear Programming (LP), Non-Linear Programming (NLP), Dynamic Programming (DP) and meta-heuristic algorithm. Each of these methods has its advantages and disadvantages. Nowadays meta-heuristic algorithms are applied frequently to solve optimization models more than before based on their unique advantages and therefore, they used in this paper (Moeini, 2019). In the following, some researches are presented for the second category.

For example, to find an optimal design of cascade spillways, Bozorg Haddad et al. (2005) used the genetic algorithm (GA) to reduce cascade spillway construction costs. The results showed that GA results had been improved in comparison with the traditional method. Bakhtyar et al. (2007) proposed another approach to minimize construction cost of cascade spillway. Their decision variables were length and height of the cascade spillway, and this problem was solved using DP. Comparison of the results of DP and Vittal and Porey (VP) method showed that construction costs were improved with 31% than VP results. Afshar and Daraeikhah (2010) used GA to optimize cascade spillway dimensions to satisfy topography and hydraulic constraints and reduce its construction costs. Finally, they compared GA results with VP results. Comparison of the results showed that when three-step cascade spillway was considered, GA results were improved with 15%, and when four-steps cascade spillway was considered, GA results were improved with 13.2% than VP results. Furthermore, Bozorg Haddad et al. (2010) used honey bee mating optimization (HBMO) for selecting the best decision variables of the design optimization problem for the cascade spillway to reduce cascade construction cost. They compared the GA results with HBMO results and showed that HBMO results were improved and HBMO algorithm were converged to the best solution faster than GA's best solution. In addition, GA and Differential Evolution (DE) algorithm were used for finding the optimal shape of labyrinth spillways in which the construction costs minimization was its objective function (Hosseini et al., 2016). The obtained results highlighted that the obtained cost of GA and DE algorithm were decreased by 6.6% and 19.3% the construction costs of a benchmark.

Reviewing the literature shows only a few research have been carried out for optimal design of cascade spillway design using effective methods such as meta-heuristic algorithms. Therefore, four meta-heuristic algorithms named GA, GSA, PSO, and ABC are used here to solve this optimization problem due to unique feature of these algorithms. This unique feature was highlighted in many research when these algorithms were used to solve different engineering optimization problems (Soghrati and Moeini, 2020; Elrehim et al., 2019; Garg, 2014, 2015, 2016, 2019; Latchoumi et al., 2019; Latif and Saka, 2019; Mahanipour and Nezamabadi-pour, 2019; Patwal et al., 2018) and therefore, they used these algorithms to solve complex optimization problem.

After presenting the introduction and researches history in the first section, in the second section, the used methods are explained. In continuous, the optimization model is given by constraints and objective function definition in the third section. In the following, some descriptions of the case study are presented in the fourth section. In the fifth section, the obtained results are presented and compared. Finally, in the sixth section, some significant remarks are presented.

2. Materials and method

In the present research, four algorithms named GA, GSA, PSO, and ABC are used to solve the optimum design of cascade spillway problem considering some assumptions. The obtained results are compared with VP's results to validate the performance of these algorithms. Therefore, in this section, some descriptions are explained about assumptions, the VP method and the algorithms.

2.1. Assumptions

Some assumptions have been considered for this research. Here a single objective optimization problem is defined in which the main objective is to minimize the construction cost of cascade spillway and other related cost has not been considered. In addition, only three and four steps conditions are considered in which other conditions lead to infeasible solution. Furthermore, probable uncertainty of variables such as design discharge was ignored and a constant valued has been considered for it. In addition, here, cascade spillway was chosen as an energy dissipater structure due to its efficient hydraulic performance and low construction cost and other alternatives have been not investigated.

2.2. Vittal and Porey (VP) method

In this method, at first, the length and height of the terminal step of cascade spillway are determined. Then, the rest length of the dam is divided to a specified number of steps (such as three or four steps) that preceding steps with same lengths are formed. Finally, the other variables should be calculated, and the general shape of the cascade spillway is shaped according to Fig.1 (Vittal and Porey, 1987).

It should be mentioned that in this method, two curves should be defined. At first, the Free Jump Hydraulic Curve (FJHC) should be defined, then, Tail Water-Rating Curve (TWRC) should be defined. These two curves can be used for calculating the height of terminal step. In this method, using the equation of FJHC Fig.2, the height of the terminal step (H_t) is calculated as Eq. (1) (Vittal and Porey, 1987):

$$H_t = \frac{g * y_{td}^4}{7.8 q_d^2} \tag{1}$$

where: y_{td} is downstream river depth at design flow, g is gravitational acceleration and q_d is designed flow in unit width of the dam.

In the following, Eqs. (2-3) are obtained to design structure considering the Bernoulli's equation for two specified sections, continuous equation and Froude number formula. It is worth noting that for simplification of Eqs. (2-3), dimensionless parameters such as p_* and q_* are used in which they are described in Eqs. (4-5) (Vittal and Porey, 1987):

$$\frac{1}{q_*} = (0.5 Fr_1^{\frac{4}{3}} + Fr_1^{\frac{-2}{3}} - \frac{1}{2^{\frac{1}{3}} * c^{\frac{2}{3}}})^{\frac{3}{2}} \tag{2}$$

$$P_* = \frac{Fr_1^2}{2} - \frac{Fr_1^{\frac{2}{3}}}{2^{\frac{1}{3}} * C^{\frac{2}{3}}} + 1 \tag{3}$$

$$P_* = \frac{P}{y_1} \tag{4}$$

$$q_* = \frac{q_d}{g^{0.5} * P^{1.5}} \tag{5}$$

where: Fr_1 is Froude number, c is flow coefficient, P is the height of the terminal step (or H). In this method, q_* is calculated using Eq. (5) and therefore, Fr_1 can be defined using Eq. (6) with try and error method.

Then, then first hydraulic depth (y_1) is calculated for the stilling basins of the terminal step (Vittal and Porey, 1987):

$$Fr_1 = \frac{V_1}{\sqrt{g * y_1}} \tag{6}$$

By calculating the first hydraulic jump, the conjugate depth can be calculated as Eq. (7) (USBR, 1985):

$$\frac{y_2}{y_1} = 0.5(\sqrt{1 + 8Fr_1^2} - 1) \tag{7}$$

where v_1 is the initial velocity before the hydraulic jump, and other parameters were defined before. By determining the conjugate depths, length of the terminal step's stilling basin (L_t) can be calculated as Eq. (8) (USBR, 1985):

$$L_t = 4.25 y_2 \tag{8}$$

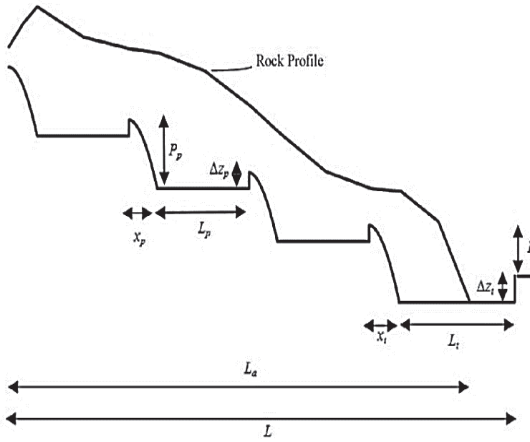

Fig. 1. Schematic cascade spillway structure (Vittal and Porey, 1987)

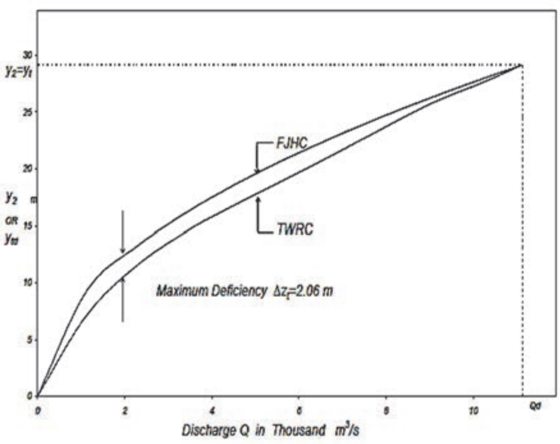

Fig. 2. *FJHC* and *TWRC* curves for terminal step (Vittal and Porey, 1987)

By comparing two curves of Fig.2, the maximum difference between the downstream elevation and second hydraulic depth (Δz_t) is determined for perfect hydraulic jump occurrence, and therefore the bottom elevation of stilling basin in the terminal step should be descended as much as Δz_t. So, the modified height of the terminal step is calculated as Eq. (9) (Vittal and Porey, 1987):

$$P_t = H_t + \Delta z_t \tag{9}$$

where: P_t is modified height of the terminal step. In the following, total head (h_{0d}) can be calculated as Eq. (10) at design flow condition (Vittal and Porey, 1987):

$$h_{0d} = \left(\frac{q_d}{c\sqrt{2g}}\right)^{\frac{2}{3}} \tag{10}$$

where: all the parameters were defined before. When the total head is calculated, horizontal chute length of the terminal step (x_t) can be calculated using Eq. (11) for ogee spillway (USBR, 1985):

$$x_t = 1.455 h_{0d} * \left(\frac{P_t}{h_{0d}}\right)^{\frac{1}{1.85}} \tag{11}$$

In the following, the specified number (N) of steps is supposed, and (Eq. 12) is solved to find the height of preceding steps by try and error method with the combination of *FJHC* and the Continuity equation (Vittal and Porey, 1987):

$$P_p = \frac{H_0 - H_t}{N-1} + 1.671\frac{q_d^{0.5} * P_p^{0.25}}{g^{0.25}} - \left(\frac{q_d}{c*\sqrt{2g}}\right)^{\frac{2}{3}} + 0.179\frac{q_d}{g^{0.5} * P_p^{0.5}} \tag{12}$$

where: H_0 is the total fall height of flow, and other parameters were defined before. By determining P_p, this value is replaced instead of P in Eq. (5) and q_* is calculated for preceding steps.

Therefore, conjugate depths of the hydraulic jump for preceding steps are calculated using Eqs. (6-7) like the terminal step. In addition, stilling basin horizontal length of the preceding step (L_p) can be calculated as Eq. (13) (Vittal and Porey, 1987):

$$L_p = 6(y_2 - y_1) \tag{13}$$

It is worth noting that summation of stilling basin horizontal length of the step and horizontal chute length of the step is considered as total horizontal length of every step (L'_i). In addition, chute horizontal length of the preceding step (x_p) is calculated by replacing the P_p instead of P_t in Eq. (11).

Total of preceding stilling basin (ΔZ_p), is calculated using Eqs. (12, 14-15) (Vittal and Porey, 1987):

$$N(P_p - \Delta z_p) = H_0 - H_t \tag{14}$$

$$\Delta z_p = 1.671\frac{q_d^{0.5} * P_p^{0.25}}{g^{0.25}} - \left(\frac{q_d}{c*\sqrt{2g}}\right)^{\frac{2}{3}} + 0.179\frac{q_d}{g^{0.5} * P_p^{0.5}} \tag{15}$$

Where all the parameters were defined before. Finally, the total horizontal length of cascade spillway is calculated as Eq. (16) (Vittal and Porey, 1987):

$$L = (N-1)(x_p + L_i) + (x_t + L_t) \tag{16}$$

It is worth noting that the mentioned process should be continued until the total length of cascade spillway (L) is less or equal than the allowable length (L_a). (Fig. 3) shows briefly the description of Vittal and Porey's (VP) method.

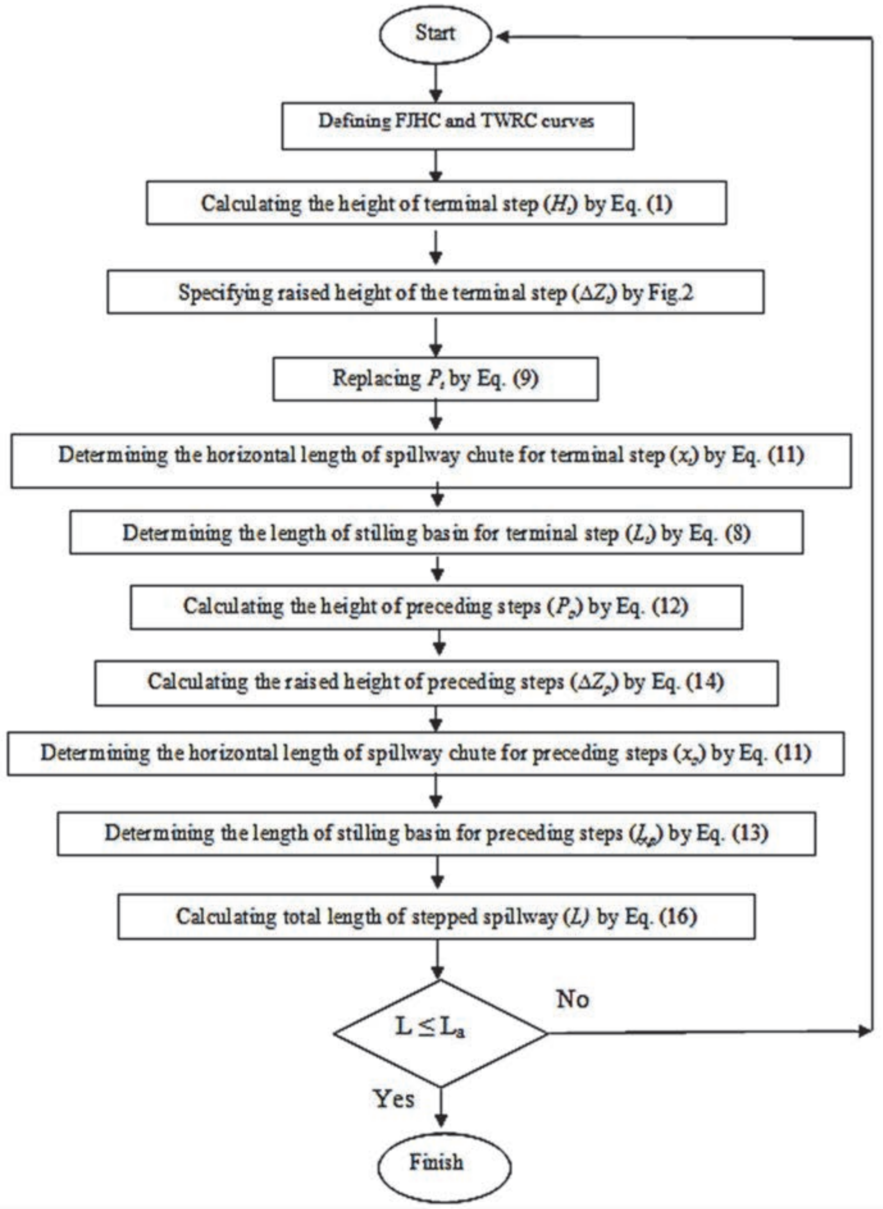

Fig. 3. The summary of the VP method

2.3. Genetic Algorithms (GA)

The genetic algorithms were suggested by Holland (1975) which is based on the natural evolution of the alive creatures. In other words, GA is an evolutionary process that the main base of evolution is inheritance.

In this algorithm, at first, initial population is made in which every single chromosome is a solution that is named the individual. Then, fitness is assigned to every individual due to the objective function of the problem in which this fitness subsequently will be used to choose superior individual. Generally, bigger fitness value leads to more selection probability for the next population generation (Holland, 1975). Three operators named selection, crossover and mutation were defined for GA in which the best solution can be obtained by setting these operators. In the following, brief descriptions of them are presented summarily.

Selection Operator: When the initial population is generated, it is necessary to choose some of them for generating next-generation named parents. This selection process is implemented using a selection operator corresponding to fitness values (Holland, 1975).

Crossover operator: When parent's chromosomes are defined, they should be compounded to produce new chromosome. This procedure can be done using a crossover operator to generate a new chromosome that has a combination of their parent's genes characteristics (Holland, 1975).

Mutation operator: Mutation is the process that replaces one gene with another gene to generate a new genetically structure. In other words, the mutation operator is an unexpected random change in the chromosome string (Holland, 1975). Fig. 4 shows a brief description of GA for finding an optimal solution.

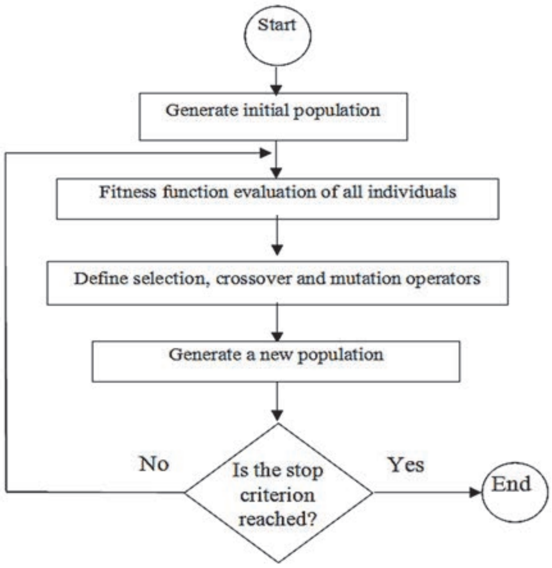

Fig. 4. The summary process of GA

2.4. Particle Swarm Optimization (PSO) algorithm

PSO algorithm is based on birds and poultry behavior (Reynolds, 1987). Here, every agent tries to move without collision to each other and keep the optimal distance from other agents. At first, the simple model of this simulation is suggested by Eberhart and Kennedy (1995).

Here, x_{iD}, v_{iD} and P_{iD} describe the location, velocity and best-visited of i th agent in dimension D. Also, the new velocity and agent location are updated as Eqs. (17-18) (Eberhart and Kennedy, 1995):

$$v_{id}^{n+1} = w * v_{id}^{n} + c_1 * r_{1,i,d}^{n} (p_{id}^{n} - x_{id}^{n}) + c_2 * r_{2,i,d}^{n} (p_{gd}^{n} - x_{id}^{n}) \quad (17)$$

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1} \quad (18)$$

where: w is the weight inertia, n expresses iteration number of algorithm, d is the problem dimension number, i is the particle number, N defines the total number of particles, c_1 is the fixed coefficient with positive value (Cognitive parameter), and c_2 is the fixed coefficient with positive value (Social parameter), $r_{1,i,d}$ and $r_{2,i,d}$ are the chance coefficients between [0,1].

Because every particle can move with the desired velocity and increasing or decreasing of the particle velocity will cause to pass the better solution and inappropriate exploration in the search space, the velocity constraint is necessary for the particle movements as Eq. (19) (Eberhart and Kennedy, 1995):

$$V_{d,\min} \le V_{i,d}^{n+1} \le V_{d,\max} \quad (19)$$

where: $V_{i,d}^{n+1}$ is the new velocity of i th particle for d th dimension, $V_{d,min}$ is the minimum velocity for d th dimension and $V_{d,max}$ is the maximum velocity for d th dimension.

According to the mentioned equations, particles are searching the space to find the optimal solution using the velocity constraint for particle movement at every iteration. The process is continued until the stop criterion is reached. Fig. 5 shows a concise description of PSO algorithm for finding an optimal solution.

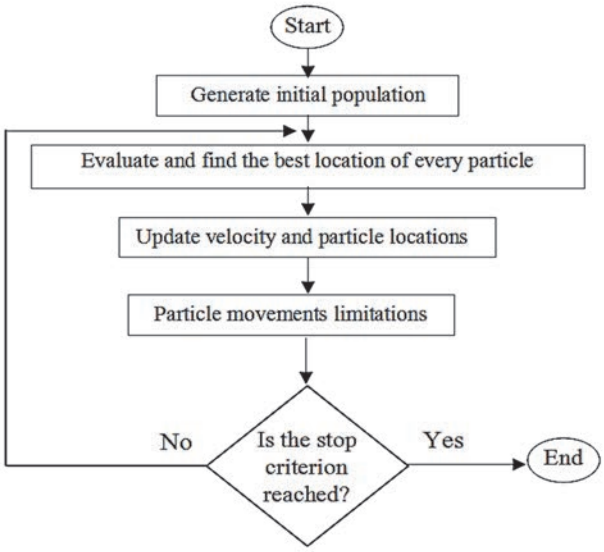

Fig. 5. The summary process of PSO algorithm

2.5. Artificial Bee Colony (ABC) algorithm

ABC algorithm is based on the social life of honey bee for discovering the food resource that is proposed by Karaboga and Basturk (2007) for solving the optimization problems. In this algorithm, the honey bee colony is assumed as the dynamic system and has collected information from surroundings and set its behavior to gather information for finding food. In this algorithm, bees are divided into three groups as employed bees (EB), onlooker bees (OB) and scout bees (SB) (Karaboga and Basturk, 2007).

In this algorithm, at first, random solutions (food resources) are generated in which they are equal to the numbers of OB. Then, the objective function which is the amount of nectar for every food resource is determined. Here, the initial solution can be generated as Eq. (20) (Naveena et al., 2015):

$$X_{ij} = X_{j,\min} + rand(X_{j,\max} - X_{j,\min}) \quad (20)$$

where: i is the food resource number, $X_{j,min}$ and $X_{j,max}$ are respectively the lowest and highest values of j th variable and $rand$ is an accidental coefficient between [0,1].

Then, the new solutions can be generated for the next iterations of the algorithm as Eq. (21) (Naveena et al., 2015):

$$v_{ij} = (X_{ij} - X_{kj}) * \Phi_{ij} + X_{ij} \quad (21)$$

where, v_{ij} presents i th new food resource of j th variable, Φ_{ij} is a chance coefficient between [0,1], and X_{ij} presents j th variable of i th food resource.

By calculating the objective function, if a new answer is a better answer, it replaces with the old answer, otherwise, the old answer remains in the memory of the algorithm. Next stage of this algorithm is related to OB in which try to find an optimal solution. The OB is moving toward the food resources with greater probability by the gained nectar information of the EB. The probability is calculated as Eq. (22) (Naveena et al., 2015):

$$P_i = \frac{Fitness(S_i)}{\sum_{n=1}^{SN} Fitness(S_n)} \quad (22)$$

where, *Fitness* (S_i) is the fitness value of answer S_i that is corresponding to the nectar amount of i th food resource and S_N is the food resources number. All the mentioned steps are maintained until the stop criterion is reached and therefore the best solution is obtained (Naveena et al., 2015).

Generally, in the ABC algorithm, the EB and the OB do the local search in the search space while the SB do the global search in the search space and therefore, the exploitation and exploration features can be better worked together for finding a good solution in this algorithm.

Fig.6 shows the summary descriptions of the ABC algorithm.

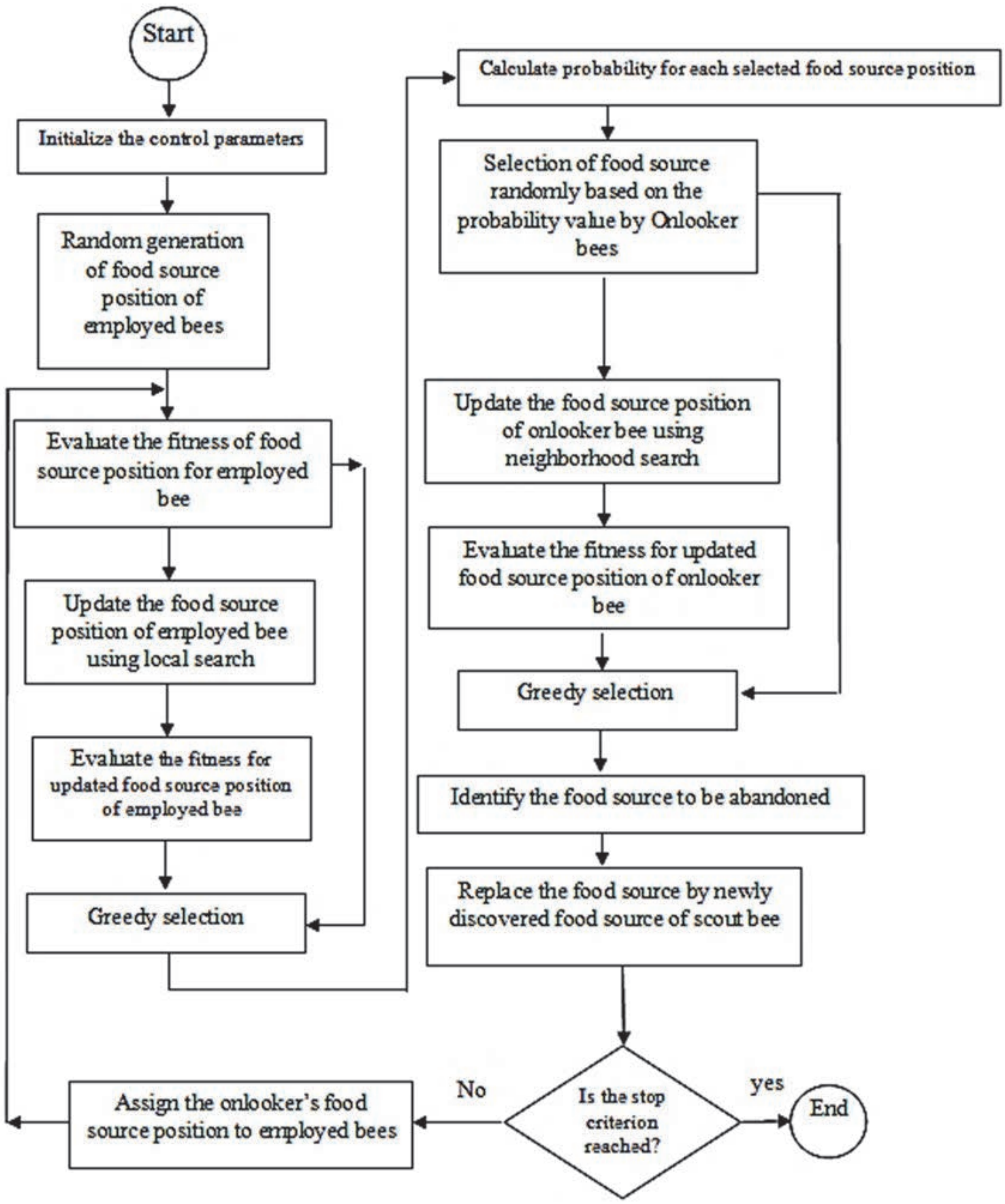

Fig. 6. The summary of ABC algorithm (Naveena et al., 2015)

2.6. Gravitational Search Algorithm (GSA)

GSA is suggested based on the natural gravitation law named Newton's gravitation law. Here, the explorer agents are the complex of objects (planets) that optimal solutions' position attracts the agents to itself like a black hole. Fitness information of every object is stored as gravitational and inertia masses. Planets reciprocal effectiveness are done by gravitation forces and related rules (Rashedi et al., 2009).

Here, $x_i^d(t)$ is the dimension of *i* th agent in *d* th dimension at iteration *t*. The inserted force of *i* th agent to *j* th agent for *d* th dimension at iteration *t*, $F_{ij}^d(t)$, is calculated as Eq. (23) (Rashedi et al., 2009):

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t)*M_{ai}(t)}{R_{ij}^{Rpower}+\varepsilon}*(x_j^d(t)-x_i^d(t)) \quad (23)$$

where, $G(t)$ is the gravitational constant at each iteration *t*, $M_{pi}(t)$ and $M_{ai}(t)$ are respectively passive and active gravitational masses of *i* th mass at iteration *t*, R_{ij} is the distant between two agents at iteration *t*, R_{power} is the power of distance between two agents and ε is a tiny number.

Finally, inserted forces on *d* th dimension of *i* th agent at iteration *t*, $F_i^d(t)$, is equal to the summation of the random coefficient of forces that *k* superior agents are inserted to *i* th mass as Eq. (24) (Rashedi et al., 2009):

$$F_i^d(t) = \sum_{j\ni k_{best},\, j\neq 1}(rand_j * F_{ij}^d(t)) \quad (24)$$

where, k_{best} is the varied collection of *k* superior agents, and $rand_j$ is a chance number between [0,1]. By calculating inserted force, the agent acceleration, $a_i^d(t)$, could be calculated, where, $a_i^d(t)$ is the *d* th dimension of *i* th agent acceleration at iteration *t*. Then, the new agent velocity and location could be determined as Eqs. (25-26) (Rashedi et al., 2009):

$$v_i^d(t+1) = rand_i * v_i^d(t) + a_i^d(t) \quad (25)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (26)$$

where, $v_i^d(t+1)$ and $v_i^d(t)$ are the new and old velocity of *i* th agent in *d* th dimension at iterations (*t+1*) *and t* respectively, $rand_i$ is an accidental coefficient between [0,1] and $x_i^d(t+1)$ is the new position at iteration (*t+1*). It is worth noting that here, gravitational constant is calculated as Eq. (27) (Rashedi et al., 2009):

$$G(t) = G_0 * \exp(\frac{\alpha * t}{T}) \quad (27)$$

where, G_0 is the initial gravitational constant, α is a positive number and *T* represent the total number of iterations.

Generally, in GSA agents are evaluated, and their movement is determined by defining the search space of problem and the parameters and updating them over the iterations. In the following, masses and gravitational constant are updated, and this process will continue until the stop criterion is reached. Fig.7 shows the brief descriptions of GSA for finding an optimal solution (Moeini et al., 2017).

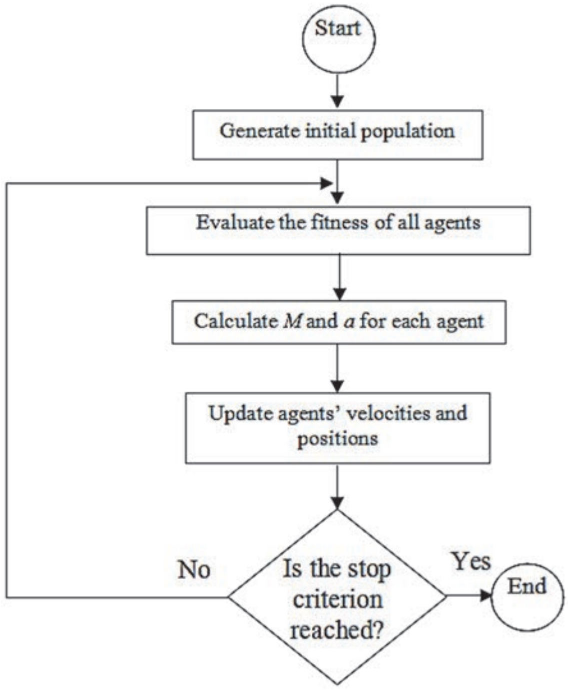

Fig. 7. The summary process of GSA (Moeini et al., 2017)

3. Mathematical optimization model of cascade spillway design

Here, the mathematical optimization model of the cascade spillway design problem is defined. Generally, every optimization problem can be modeled by defining decision variable(s), objective function and constraint(s).

Here, step heights (P_i) and horizontal length of the stilling basin (L_i) are the decision variables of the problem. For this problem, different forms of the objective function can be defined. In the present research, the objective function is defined to minimize the construction cost of the cascade spillway as Eq. (28):

$$Minimize f = \sum_{i=1}^{N}(f_1(P(i),L(i)) + f_2(p(i),L(i))) \quad (28)$$

where: *f* is the total construction costs that consist of two main costs such as excavation cost (f_1) and concrete cost (f_2) in which both of them are related to steps height and stilling basins length. Generally, the total volume of excavation and concrete are calculated using the topography condition, height steps, and stilling basins length as Eq. (29-31) considering unit width:

$$\sum_{j=1}^{L-1}[(y_j - zz_j)+(y_{j+1} - zz_{j+1})]*1*1*\frac{1}{2} = V_{e1} \tag{29}$$

$$\sum_{j=L}^{L_a}[(y_j - y_{rl})+(y_{j+1} - y_{rl})]*1*1*\frac{1}{2} = V_{e2} \tag{30}$$

$$f_1 = (V_{e1} + V_{e2})*c_1 \tag{31}$$

where: y_j is the elevation of construction side (topography) at point j, zz_j is the elevation of stilling basins at point j, z_j is the elevation of spillway and its chute at point j, y_{rl} is the bottom elevation of downstream river, L is the total length of constructed cascade spillway, L_a is the allowable length for constructing cascade spillway, V_{e1} and V_{e2} are the excavation volumes.

As the same way, concreting costs are calculated using equations as Eq. (32-34):

$$\sum_{j=1}^{L}[(z_j - zz_j)+(z_{j+1} - zz_{j+1})]*1*1*\frac{1}{2} = V_{c1} \tag{32}$$

$$\sum_{j=1}^{L}[L_i]*1*1*\frac{1}{2} = V_{c2} \tag{33}$$

$$f_2 = (V_{c1} + V_{c2})*c_2 \tag{34}$$

where: L_i is the horizontal length of cascade spillway stilling basins, V_{c1} and V_{c2} are the concreting volumes, and other parameters were defined before.

To complete the modeling of the optimization problem, all constraints of this problem should be defined. Generally, this problem has hydraulic and topographic constraints as follows. In this problem, the summation of steps heights and raised heights at the end of the stilling basins should not be greater than total fall height as Eq. (35):

$$g_1 = H_0 - (\sum_{i=1}^{N-1}(P(i) - \Delta z(i)) - (P_t - \Delta z_t)) \geq 0 \tag{35}$$

where, H_0 is the total fall height, $P(i)$ is the step height of preceding falls, Δz_t is the raised height at the end of the stilling basin in terminal step, P_t is the step height of terminal step and Δz_p is the raised height at the end of the stilling basins in preceding steps. Also, first constraint (Eq. 35) is necessary to observe for the terminal step as Eq. (36):

$$g_2 = P_t - (H_t + \Delta z_t) \geq 0 \tag{36}$$

where, all the parameters were defined before. In addition, the horizontal length of the stilling basins should be smaller than allowable length as Eq. (37):

$$g_3 = L_a - \sum_{i=1}^{N}(L(i) + x(i)) \geq 0 \tag{37}$$

When algorithms starts to find optimal solutions for the height of steps, P, and horizontal length of the stilling basin, L, a hydraulic limitation should be handled the solution values. Therefore, the minimum and maximum allowable heights for the steps of the cascade spillway are suggested by *USBR* criterion as follows: P_{min}=*30.96 m*, P_{max}=*92.58m.*

In addition, the minimum horizontal length for the stilling basin was proposed by Eqs. (8-13). In addition, hydraulic constraints are considered for this problem to form a complete hydraulic jump as Eqs. (38-40):

$$g_4 = P_{\max} - P(i) \geq 0 \tag{38}$$

$$g_5 = P(i) - P_{\min} \geq 0 \tag{39}$$

$$g_6 = L_i - l_{i,\min} \geq 0 \tag{40}$$

where, P_{max} is the maximum height of the step, P_{min} is the minimum height of the step, $l_{i,min}$ is the minimum horizontal length for the stilling basin which has chosen by the *USBR* criterion and L_i is the calculated length for stilling basins.

Optimal design problem of cascade spillway is a constrained optimization problem. Generally, different methods are used to handle problem constraints. One of the usual methods is the penalty coefficient method. In this approach, when the solution is infeasible, the amount of constraints violation is calculated and multiplied to the fix penalty coefficient and summated with the original objective function value as Eq. (41):

$$f_p = \begin{cases} f & ,if \quad solution \quad feasible \\ f + M * \sum_{i=1}^{G} CSV_i & ,otherwise \end{cases} \tag{41}$$

where, f is the original objective function value, f_p is the penalized objective function value, CSV_i is the summation of constraints violation for i th constraint, M is the penalty coefficient, and G is the total number of constraints.

4. Case study

In this paper, information of cascade spillway for Tehri dam in India are presented in which it was used as a case study. Tehri Dam is a multi-purpose rock and earth-fill embankment dam on the Bhagirathi River near Tehri in Uttarakhand, India, and its spillway located beside dam's body. Tehri dam in one of the largest dams and the fifth tallest dams (260 m height) which was constructed 200 miles north east of Delhi and has the potential of 100 megawatts energy generation. In the present research, all uncertainty such as flood discharge design were ignored because of problem simplification (IRN, 2002).

The most important information of Tehri dam such as the total width of the crest (w), cascade spillway head elevation (El_c), flood discharge design (Q_d), crest length (C_r), bed river elevation in downstream (El_b), total fall height (H_0), tailwater depth for flood design (y_{td}) and available horizontal length (L_a) is presented in Table 1, Fig. 8.

Table 1. Tehri dam information (Vittal and Porey, 1987)

w (m)	*El_c (m)*	*Q_c (m^3/s)*	*C_r (m)*	*El_b (m)*	*H_0 (m)*	*y_{td} (m)*	*L_a (m)*
95	818	11000	80	600	218	29.2	778

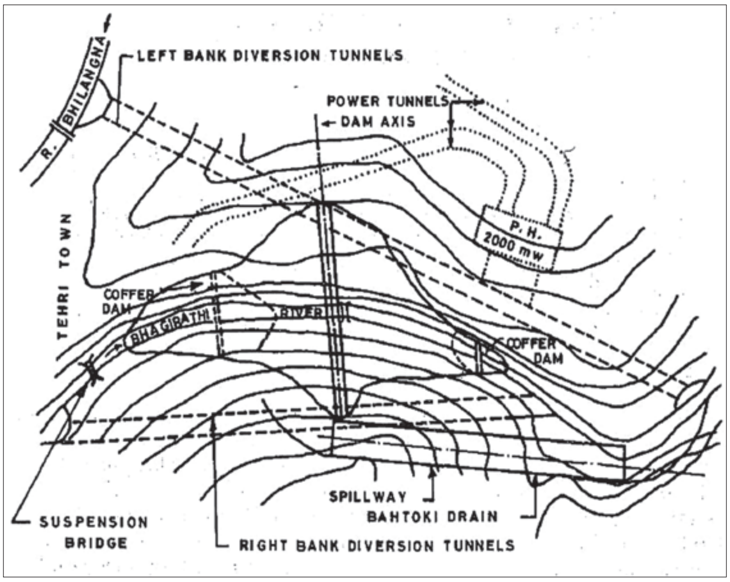

Fig. 8. The Tehri dam plan (Vittal and Porey, 1987)

5. Results and discussion

Here, all the obtained results are presented and compared with each other to evaluate the effectiveness and robustness of proposed method. At first, the cascade spillway is designed using the VP method. The only allowable conditions for the specified cascade spillway are three and four-stepped conditions because hydraulic jump has formed in stilling basins completely (the hydraulic constrains have been satisfied) and constructed total length is smaller than allowable length. The obtained parameters values are presented in Table 2 using VP method.

In both conditions, the horizontal length of terminal step chute (x_t) and the horizontal length of stilling basins in the terminal step (L_t) are 49.16m and 125.04m, respectively. The excavation costs are 23100 units per cubic meter, and the concreting costs are 180000 units per cubic meter. Therefore, the total construction costs are 1094 and 1194 million units considering three or four stepped conditions respectively using the VP method.

The algorithms have many parameters in which the sensitivity analysis should be done to obtain the best values of them in which the details are not presented here to avoid lengthy paper. All the algorithms have been solved using 100,000 function evaluation. All the results such as the minimum, maximum and average solution cost values (million units), normalized standard deviation (*StdN*) and the number of feasible solutions (*NFS*) in 10 runs are presented in Table 3 and Table 4 for three and four steps conditions, respectively. Fig. 9 and Fig. 10 show the convergence curve of the minimum solution cost values obtained using four algorithms for both conditions (*population*=25 and *iteration*=400).

In the following, calculated dimensions of the cascade spillway and stilling basins for the best solution of each used algorithm at three and four-stepped conditions are presented in Table 5. In this Table, P_i is the height steps, L_i is the horizontal length of stilling basins, x_i is the horizontal length of cascade spillway chute, and Δz_i is the raised height of preceding steps and the sinking height of the terminal step. Also, all GSA's, PSO's and ABC's obtained results are better than GA's results in both three-stepped and four-stepped conditions.

When the three-stepped condition is considered, the best solution of GSA, PSO, and ABC are improved with respectively 0.3%, 1.8% and 1.7% than the best solution of GA. Furthermore, when the four-stepped condition is considered, the best solution of GSA, PSO and ABC are improved with respectively 1.9%, 2.3% and 1.8% than the best solution of GA. In addition, for both three and four-stepped conditions best results are obtained using PSO algorithm due to the better interaction of exploitation and exploration features.

Finally, the comparison of the results presented in Table 3 and Table 4 shows that when the three-stepped condition is considered, the best solution cost values of GA, GSA, PSO and ABC algorithms are improved with respectively 16.16%, 16.4%, 17.7% and 17.63% than VP results. In the same manner, when the four-stepped condition is considered, the best solution cost values of GA, GSA, PSO and ABC algorithms are improved with respectively 14.5%, 16.1%, 16.45% and 16.05 %than VP results.

Table 2. Parameters values Tehri spillway using VP method

N	$P_p(m)$	$x_p(m)$	$L_i(m)$	$\Delta z_p(m)$	$L(m)$
3	93.55	58.15	175.39	17.8	641.28
4	65.75	48.06	156.61	15.25	788.20

Table 3. The obtained results for three-stepped condition using all proposed algorithms

Algorithm	*Population*	*Iteration*	*NFS*	*Minimum costs*	*Maximum costs*	*Average Costs*	*StdN*
GA	25	4000	10	**1001**	1054.9	1023.8	0.0219
	50	2000	10	**1001**	1063.8	1030.4	0.0208
	100	1000	10	1006.5	1062	1028.1	0.0190
	200	500	10	1008.7	1031.2	1016.7	0.0079
	250	400	10	1004.4	1034.6	1022.8	0.0097
GSA	25	4000	10	**998.2**	1013.7	1006.6	0.0047
	50	2000	10	1013.1	1037.5	1027.1	0.0081
	100	1000	10	1028.9	1049.1	1038.4	0.0069
	200	500	10	1028.4	1063.8	1052.2	0.0126
	250	400	10	1041.9	1065.9	1058.9	0.0073
PSO	25	4000	10	983.2	1027	1013	0.0186
	50	2000	10	**982.7**	1031.1	1006.9	0.0232
	100	1000	10	983.5	1065.5	1009.3	0.0305
	200	500	10	983.3	1070	1016.4	0.0348
	250	400	10	983.3	1033.8	1008.1	0.0253
ABC	25	4000	10	**983.5**	994	984.5	0.0031
	50	2000	10	985.6	1005	989.3	0.0059
	100	1000	10	984.4	987	985.4	0.0010
	200	500	10	987.2	999.7	993.1	0.0038
	250	400	10	985.8	991.6	987.6	0.0023

Table 4. The obtained results for four-stepped condition using all proposed algorithms

Algorithm	*Population*	*Iteration*	*NFS*	*Minimum costs*	*Maximum costs*	*Average Costs*	*StdN*
GA	25	4000	10	945.5	1002.5	971.4	0.0210
	50	2000	10	**935.4**	1009.4	967.8	0.0217
	100	1000	10	945.5	1010.9	963.7	0.0192
	200	500	10	939.8	1003.9	963.4	0.0183
	250	400	10	937.5	975	955.5	0.0146
GSA	25	4000	10	918.4	956.8	937.7	0.0149
	50	2000	10	920.5	955	934.2	0.0145
	100	1000	10	918.1	940	928.8	0.0075
	200	500	10	**917.9**	950.4	934	0.0100
	250	400	10	921.2	952.5	935	0.0107
PSO	25	4000	10	**914**	992	942	0.0339
	50	2000	10	918.2	1007	959.9	0.0305
	100	1000	10	918.2	1004.1	957.1	0.0275
	200	500	10	914.3	1010.6	938.5	0.0351
	250	400	10	916.5	998.3	962.8	0.0301
ABC	25	4000	10	920	967	939.1	0.0148
	50	2000	10	918.8	950.6	932.3	0.0126
	100	1000	10	922.4	974.9	939	0.0160
	200	500	10	**918.4**	961.9	936.5	0.0151
	250	400	10	927.5	981.9	942.9	0.0184

6. Conclusions

In the present study, four meta-heuristic algorithms named GA, GSA, PSO and ABC were used to minimize the construction cost of the cascade spillway which includes excavation and concrete costs. The cascade spillway of Tehri dam in India was selected as the case study. Here, at first, the mathematical formulation was presented and solved using a traditional method such as VP method and the meta-heuristic algorithms. Comparison of the obtained results showed that VP method was not an effective method to solve this problem and only can find sub-optimal solutions using this method. In addition, in VP method, when three steps was considered, the total length of constructed cascade (788.2 *m*) did not satisfy the total available length (L_a=778 *m*).

This fact motivated the author to use meta-heuristic algorithms due to unique feature of these algorithms.

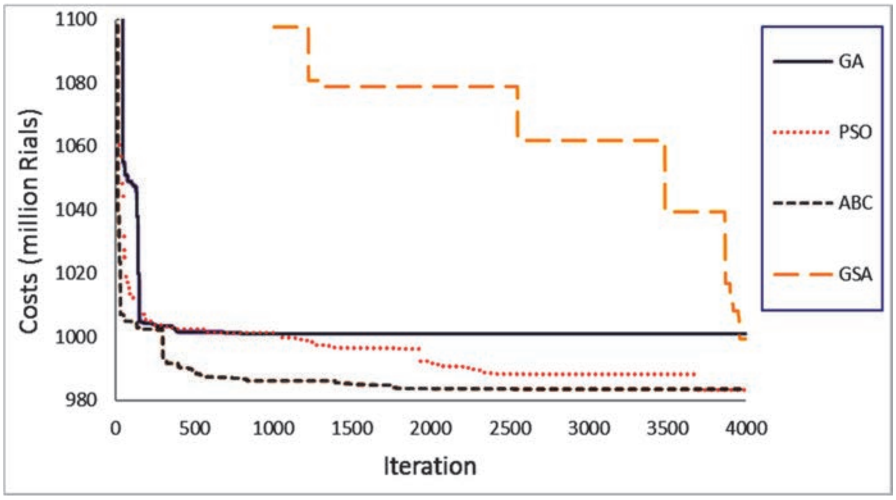

Fig. 9. Convergence curve of the solution obtained using proposed algorithms for three-stepped condition

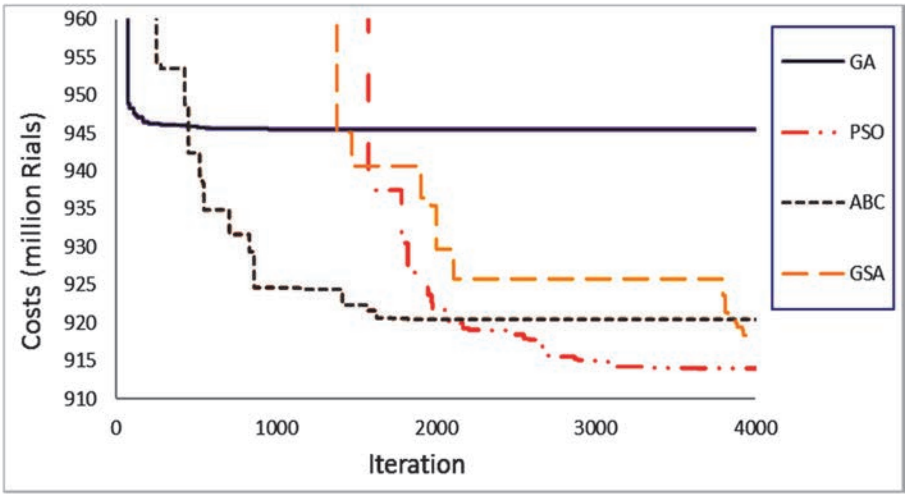

Fig. 10. Convergence curve of the solution obtained using proposed algorithms for four-stepped condition

Table 5. The dimensions of cascade spillways obtained using proposed algorithms

State	*Algorithms*	*Step number*	P_i *(m)*	L_i *(m)*	x_i *(m)*	Δz_i *(m)*
3 steps	**GA**	1	68.10	293.95	48.96	15.84
		2	92.37	185.81	57.74	17.69
		3	92.40	133.79	57.74	2.06
	GSA	1	73.84	300.65	51.16	16.05
		2	86.90	176.77	55.86	17.23
		3	92.24	134.39	57.69	2.06
	PSO	1	68.08	305.83	48.96	15.48
		2	92.40	173.90	57.74	17.70
		3	92.40	133.79	57.74	2.06
	ABC	1	68.15	304.89	48.98	15.49
		2	92.33	173.93	57.72	17.69
		3	92.40	133.97	57.74	2.06
4 steps	**GA**	1	38.94	135.10	36.20	11.92
		2	59.38	153.87	45.47	14.55
		3	78.80	166.46	52.98	16.52
		4	85.56	131.37	55.39	2.06
	GSA	1	30.93	136.92	31.96	10.62
		2	59.77	152.80	45.63	14.60
		3	78.35	165.14	52.82	16.48
		4	92.40	134.01	57.74	2.06
	PSO	1	30.90	142.41	31.95	10.61
		2	45.11	140.63	39.19	12.80

		3	92.40	173.60	57.74	17.70
		4	92.40	134.05	57.74	2.06
	ABC	1	30.90	132.18	31.94	10.61
		2	56.27	156.43	44.17	14.20
		3	81.75	167.06	54.05	16.78
		4	92.40	133.68	57.74	2.06

Using these algorithms leads to near optimal solution with allowable computational time and cost. Comparison of the results showed that the obtained algorithm's results have been considerable improved rather than VP results due to efficient interaction between exploration and exploitation mechanisms of each algorithm. In addition, to evaluate the performance of the algorithms, the results of algorithms were compared to each other.

Comparison of the results showed the efficiency and affectivity of meta-heuristic algorithms to solve highly constrained large scale optimization problems. Finally, for future works, this approach can be used for other types of spillways and new meta-heuristic algorithms can also be used to solve this problem. In addition, some model limitation can improved by adding some uncertainty issues such as variable design discharge.

Furthermore, other objective function can be used for this problem and multi objective form of the problem can be defined.

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