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MATHEMATICAL MODEL FOR PARTICLE MOTION APPLIED ON A MANURE SPREADING APPARATUS USED IN ENVIRONMENTALLY FRIENDLY TECHNOLOGY

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Abstract

It is acknowledged that organic fertilizers and wastes, such as animal slurries/manure from intensive farm enterprise, sewage sludge, poultry litter are and will continue to be spread on agricultural land and provide beneficial nutrients to crops. However, many of these materials which are also potentially polluting if not properly managed can pose a risk to groundwater and surface water quality. Generally, their distribution on the soil is performed with specialized machines to reduce the risk of overdosing (pollution) or underdosing (inefficiency) with material. For these machines to be properly dimensioned it is necessary to create mathematical models that take into account the factors that influence the distribution of the material on the soil and which can be validated experimentally.

This paper presents a mathematical model for organic waste (manure) movement containing second and first derivatives, based on force equilibrium. For simplicity, we divided the path of the material point into two parts: the first part consists of curved surface (helicoid), the second part consists of a parabolic one described in the air. The two movements are studied separately considering that particle position and speed at the end of motion should be the initial conditions of motion for the second path. There are taken into consideration the relations between design parameters of the distribution machine and the material used, relations that have logical-mathematical and theoretical foundations in classical mechanics. Also, an equation that is used to calculate the necessary time for the manure to reach the soil is given together with different working hypotheses.

Keywords: manure spreader, mathematical model, distribution, spreading, uniformity

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1. Introduction

Different studies have shown that manure application can have a significant impact on the chemical, physical and biological properties due to the organic matter contained in the manure (ASAE, 2005; FAO, 2011; Brown, 2013; Shi et al., 2016). Manure has the capacity to create water stable aggregates in the soil that increase infiltration, porosity and water holding capacity and decrease soil compaction and erosion. Manure application also reduces the energy required for tillage and root penetration, increases exchange and soil buffering capacity that enable manure-treated soils to retain nutrients and other chemicals for longer periods of time. As has recently been shown soil erosion and surface water runoff can be reduced by application of manure (MG, 2015; Verbree et al., 2010).

Other aspect referring to land application of animal manures are the potentially negative consequences of rising atmospheric CO_2 on the global climate and ammonia volatilization that is the

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major N loss for surface applied manures (Ge et al., 2016; Webb et al., 2010).

The spreading equipment supplied by the most important manufacturers currently available on the market has shown several methods of manure spreading:

- vertical beaters manure spreading, that is the most common method of spreading;
- horizontal beaters manure spreading that is the oldest method used;
- wide – angle spreading devices that are very used in the present.

The most important problem that spreading machines have is the impossibility of adjusting the spreading width during operation. That is why the coefficient of variation (CV) is very large over the entire width of spreading, and the marginal swats must be overlapping to have a good surface uniformity.

A mathematical model will indicate the optimal machine parameters correlated with manure properties - mathematical and theoretical foundations in classical mechanics; the model allows to establish the optimal rotational speed of distribution, the influence of friction coefficient between material and distribution apparatus, the influence of the initial position of the particle on the disk, the inclination angle of the distribution apparatus, the influence of distribution apparatus size etc. Most studies are referring to the motion of chemical fertilizer particle on spinning disks equipped with pitched straight or curved vanes (Antille, et al., 2015; Grift and Kweon, 2006; Maldaner et al., 2016).

A few studies (Landry et al., 2004, 2005, 2006; Grift and Kweon, 2006) have mentioned the manure pieces in motion and the relations between material and spreading equipment. The first condition required to obtain an even distribution of the product on soil surface is to correlate manure properties with the characteristics of the land application equipment.

Physical properties of solid and liquid manures and their effects on the performance of spreading machines were studied by Malgeryd and Watterberg (1996). Their results and conclusions represent the basis for a European standard for testing methods and requirements for manure and slurry spreaders. Bulk density, stacking ability (manure consistency), comminution resistance, heterogeneity and dry matter content were considered important. Landry et al. (2004) studied the physical and rheological properties of manure products that influence the performance of handling and land application equipment. They found that total solids concentration, bulk density, particle size distribution, friction characteristics and shearing properties had the most influence on the performances of manure handling and land application equipment. Landry et al. (2005) studied the performances of different conveying systems for manure spreaders and the effect of the hopper geometry on material flow with a prototype land applicator with both a scraper conveyor and a four augers conveyor. Authors (Landry et al., 2006) developed a numerical modelling of the flow of organic fertilizers in land application equipment using the discrete element method (DEM) and the numerical modelling method of computational fluid dynamics (CFD). Thus, they were able to simulate the spreading operations for a spreader with two vertical beaters illustrating the surface of the ground distribution of compost particles, the transversal and longitudinal distribution, flow rate, time of total unloading time and the specific energy consumption for different gate positions.

In Romania there were also concerns in this field, and a series of studies were done to improve the spreading equipment, work performance indicators especially quality parameters and energetic indices (Popa et al., 2008; Popa et al., 2009). Fertilizer spread pattern is linked to machine characteristics, particle properties and, in practice, the quality of the spreading is highly dependent on the setting of the implements. In order to study the impact of different parameters on spreading, several authors have approached the problem of particle dynamics.

2. Material and methods

The most used mathematical model for spreading the fertilizer particles is that of material point that first moves on a fixed surface having the guiding role (rotors helical conveyor), and after it leaves this surface it moves freely in gravity field, in the air. Obviously, the approximation of a fertilizer particle as a material point is accompanied by an abstract presentation/error that automatically leads to other errors in comparison with reality. Manure particles spread don't have regular shape, they have an irregular surface (between 1mm and 60 mm), different mass and can adhere between them as large groups (by cohesion). Though, at different optimum humidity and certain atmospheric conditions it can be admitted that material particles spread behave as material points.

As a result of arguments above, the equations of material point dynamics are used for mathematical modelling. For simplifying, we divided the trajectory of material point (fertilizer point) in two parts: the first part is the curve described on rotor helical surface, the second one being the curve described by particle in the air.

One manure particle from rotor surface moves according to dynamics laws (Roşca and Ion, 1981; Cândea et al., 2003). Due to the forces exerted upon particles as a result of rotor's rotation, the particle moves on rotor's surface and then it moves in the air and reaches the soil. In order to simplify the model, air resistance was neglected, taking into account that it would affect the particle height and its forward movement.

Movement of one manure particle in absolute space, where the manure is spread is a complex movement made of the following movements:

- particle movement on disk, after being dislodged from the matter to be spread;
- particle movement in the air after being dislodged from the disk;

Fig. 1 is the graphic representation of distribution apparatus mounted on machine MG5 (Popa, 2005) containing four rotors, each of them with three helical surfaces that take over the material from machine body. It is specified the rotors orientation, axes system *xoyz* from figure upper side, that is a system of parallel axes with absolute system *OXYZ* and local system on coil $o_1x_1y_1z_1$. Axes o_1x_1 and o_1y_1 of reference local systems are represented on helical surface drawings (Fig. 1).

Solutions of particle's motion on the disk will be given for each detaching area from the helical surface. Initial solutions, namely the manure particle's ones on helical surface will be given in polar coordinates and then passed to Cartesian coordinate systems. By solving the equation systems on helical surface, are obtained coordinates in r and θ, till the particle detachment, that pass afterwards to Cartesian coordinates $o_{ij}x_{ij}y_{ij}z_{ij}$, $i=1,...,4$ $j=1,2,3.$ Then, particle movement in gravitational field is solved, aiming to eventually determine the coordinates of trajectories crossing point with plan OXYZ (soil).

2.1. Description of helical surfaces

We shall consider the absolute, fixed coordinate system OXYZ, located on soil against which the machine movement takes place and depending on which will be calculated the distance reached by particles thrown from rotor's surface.

Each rotor will have its own coordinate system Oxyz to which the three continuous coils are placed. At its turn, each helical surface will have a reference system made of unit vectors $\vec{i}_r$, $\vec{i}_\theta$, and $\vec{n}$, which multiplied by radial and angular coordinates will give speed and acceleration on radial, angular and normal direction to surface.

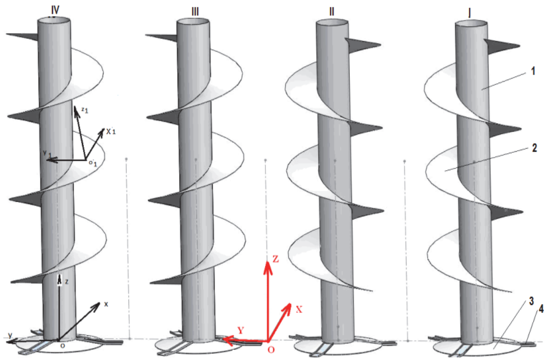
It is considered the local coordinate system of a coil, $o_1x_1y_1z_1$ (Fig. 1), and are described the parameter equations of helical surface with pitch p, with left winding (y symbol is negative), respectively right winding (y symbol is positive) (Atanasiu, 1973; Cândea et al., 2003; Roşca and Ion, 1981):

$$\begin{cases} x = r\cos\theta \\ y = \pm r\sin\theta \\ z = \mathrm{K}(r) + p\theta, p > 0, p \in R \end{cases} \quad (1)$$

where r is radial coordinate; Θ - angular coordinate; p - helical coil pitch; *K(r)* - function that shapes the helical crown (surface radial deformation is zero for classic plane helical surface).

For exemplification, we shall consider the helical surface with right winding. *K(r)* function is considered to be equal to zero because the surface radial deformation is zero for classic plane helical surface. Helical surface equations become as given by Eq. (2):

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = p\theta, p > 0, p \in R \end{cases} \quad (2)$$



(a)

(b)

Fig. 1. (a) Spreading apparatus of machine MG5; (b) Spreading machine MG5: 1) axis; 2) helical coil; 3) disk; 4) blade

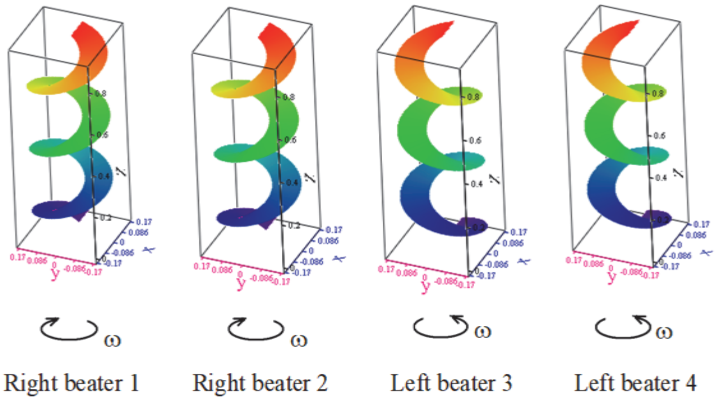


Fig. 2. Generating the helical surfaces of the spreading apparatus

We take as a material point *M* placed on surface *S*. Friction force F_f, centrifugal force F_c, gravity force *G* and normal force *N*, act upon it. The particle moves with speed v_t on a trajectory included in helical surface, having a right tangent coloured in orange, from Fig. 2. Centrifugal force is applied on the radius connecting the cylinder centre to particle centre (on radial direction).

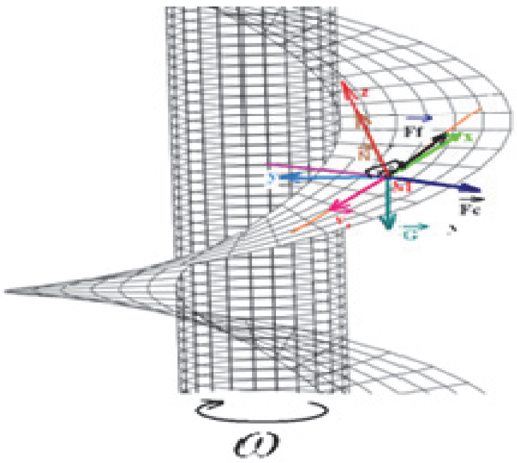


Fig. 3. Representation of forces, acting upon the point M, located on helical surface S

The material point M motion equation on surface S according to the fundamental law of dynamics (Newton's 2nd law), against a reference inertial system considered as fixed (XOYZ) to which the material point M moves (Fig. 3), is in accordance with (Atanasiu, 1973; Cândea et al., 2003; Roşca and Ion, 1981):

$$m\vec{a} = \vec{F}_c + \vec{G} + \vec{N} + \vec{F}_f \quad (3)$$

where: $\vec{F}_c$ is the centrifugal force; $\vec{G}$ - particle gravity force; $\vec{N}$ - surface reaction force; $\vec{F}_f$ - the friction force between particle and surface.

The manure piece speed is given by Eq. (4) and acceleration by Eq. (5):

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} \quad (4)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} \quad (5)$$

Coriolis acceleration and transport acceleration are negligible in comparison with gravitational acceleration, as they have very small values according to Duhovnik et al. (2004). Three axes of coordinates, perpendicular between them, x, y, z, are chosen. If *i, j* and *k* are unit vectors of the three coordinate axes (on direction/sense of increasing coordinates *x, y* and *z*) then the position vector *r* may be written as given by Eq. (6):

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} = r\cos\theta\vec{i} + r\sin\theta\vec{j} + p\theta\vec{k} \quad (6)$$

Position vector is expressed by Eq. (7):

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{\sqrt{r^2 + p^2\theta^2}} = \frac{r\cos\theta}{\sqrt{r^2 + p^2\theta^2}}\vec{i} + \frac{r\sin\theta}{\sqrt{r^2 + p^2\theta^2}}\vec{j} + \frac{p\theta}{\sqrt{r^2 + p^2\theta^2}}\vec{k} \quad (7)$$

Velocity vector is determined by derivation of the position vector according to Eq. (8), and the acceleration vector according to Eq. (9):

$$\dot{\vec{r}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} = (\dot{r}\cos\theta - r\dot{\theta}\sin\theta)\vec{i} + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)\vec{j} + p\dot{\theta}\vec{k} \tag{8}$$

$$\begin{gathered}\ddot{\vec{r}} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} = (\ddot{r}\cos\theta - 2\dot{r}\dot{\theta}\sin\theta - r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta)\vec{i} - \\ -(\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta + r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta)\vec{j} + p\ddot{\theta}\vec{k}\end{gathered} \tag{9}$$

Vectors of surface in helical surface plan are given by Eq. (10):

$$\begin{gathered}\frac{\partial\vec{r}}{\partial r} = \cos\theta\vec{i} + \sin\theta\vec{j} \\ \frac{\partial\vec{r}}{\partial\theta} = -r\sin\theta\vec{i} + r\cos\theta\vec{j} + p\vec{k}\end{gathered} \tag{10}$$

We define the unit vectors of helical surface on radial angular and normal direction (Eqs. 11, 12):

$$\vec{i}_r = \frac{\frac{\partial\vec{r}}{\partial r}}{\left|\frac{\partial\vec{r}}{\partial r}\right|} = \cos\theta\vec{i} + \sin\theta\vec{j} \tag{11}$$

$$\vec{i}_\theta = \frac{\frac{\partial\vec{r}}{\partial\theta}}{\left|\frac{\partial\vec{r}}{\partial\theta}\right|} = \frac{-r\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{r\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{p}{\sqrt{r^2+p^2}}\vec{k} \tag{12}$$

We define normal to surface as the product between the 2 vectors $\vec{i}_r$ and $\vec{i}_\theta$ (Eq. 13):

$$\vec{n} = \vec{i}_r \times \vec{i}_\theta = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 0 \\ \frac{-r\sin\theta}{\sqrt{r^2+p^2}} & \frac{r\cos\theta}{\sqrt{r^2+p^2}} & \frac{p}{\sqrt{r^2+p^2}} \end{pmatrix} = \frac{p\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{p\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{r}{\sqrt{r^2+p^2}}\vec{k} \tag{13}$$

2.2. Finding the movement equations

Motion equation projection on the local mark includes the following:
- Speed coordinate on radial direction (Eq. 14):

$$\dot{\vec{r}}\cdot\vec{i}_\theta = \left[(\dot{r}\cos\theta - r\dot{\theta}\sin\theta)\vec{i} + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)\vec{j} + p\dot{\theta}\vec{k}\right]\cdot\left(\frac{-r\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{r\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{p}{\sqrt{r^2+p^2}}\vec{k}\right) = \sqrt{r^2+p^2}\cdot\dot{\theta} \tag{14}$$

$$\Rightarrow v_r = \dot{r}$$

- Speed coordinate on angular direction (Eq. 15):

$$\dot{\vec{r}}\cdot\vec{i}_\theta = \left[(\dot{r}\cos\theta - r\dot{\theta}\sin\theta)\vec{i} + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)\vec{j} + p\dot{\theta}\vec{k}\right]\cdot\left(\frac{-r\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{r\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{p}{\sqrt{r^2+p^2}}\vec{k}\right) = \sqrt{r^2+p^2}\cdot\dot{\theta} \Rightarrow v_\theta = \sqrt{r^2+p^2}\cdot\theta \tag{15}$$

- Speed coordinate on normal direction (Eq. 16):

$$\begin{gathered}\dot{\vec{r}}\cdot\vec{n} = \left[(\dot{r}\cos\theta - r\dot{\theta}\sin\theta)\vec{i} + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)\vec{j} + p\dot{\theta}\vec{k}\right]\cdot\left[\frac{p\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{p\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{r}{\sqrt{r^2+p^2}}\vec{k}\right] = 0 \\ \Rightarrow v_n = 0\end{gathered} \tag{16}$$

- Speeds coordinate on normal coordinate is zero.
- Speed in Cartesian description and in coordinate system of helical surface shall be (Eq. 17):

$$\dot{\vec{r}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} = v_r\vec{i}_r + v_\theta\vec{i}_\theta = \dot{r}\vec{i}_r + \sqrt{r^2+p^2}\dot{\theta}\vec{i}_\theta \tag{17}$$

Similarly, we shall define the accelerations as follows:

- Radial acceleration (Eq. 18):

$$a_r = \ddot{\vec{r}} \cdot \vec{i}_r = \left[(\ddot{r}\cos\theta - 2\dot{r}\dot{\theta}\sin\theta - r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta)\vec{i} - (\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta + r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta)\vec{j} + p\ddot{\theta}\vec{k})\right] \cdot$$
$$\cdot(\cos\theta\vec{i} + \sin\theta\vec{j}) = \ddot{r} - \dot{r}\dot{\theta}^2 \qquad \Rightarrow a_r = \ddot{r} - \dot{r}\dot{\theta}^2 \tag{18}$$

- Angular acceleration (Eq. 19):

$$a_\theta = \ddot{\vec{r}} \cdot \vec{i}_\theta = \left[(\ddot{r}\cos\theta - 2\dot{r}\dot{\theta}\sin\theta - r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta)\vec{i} - (\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta + r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta)\vec{j} + p\ddot{\theta}\vec{k})\right] \cdot$$
$$\cdot(\frac{-r\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{r\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{p}{\sqrt{r^2+p^2}}\vec{k}) = \sqrt{r^2+p^2}\ddot{\theta} + \frac{2r\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}} \qquad \Rightarrow a_\theta = \sqrt{r^2+p^2}\ddot{\theta} + \frac{2r\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}} \tag{19}$$

- Normal acceleration (Eq. 20):

$$a_n = \ddot{\vec{r}} \cdot \vec{n} = \left[(\ddot{r}\cos\theta - 2\dot{r}\dot{\theta}\sin\theta - r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta)\vec{i} - (\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta + r\ddot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta)\vec{j} + p\ddot{\theta}\vec{k})\right] \cdot$$
$$\cdot\frac{p\sin\theta}{\sqrt{r^2+p^2}}\vec{i} + \frac{p\cos\theta}{\sqrt{r^2+p^2}}\vec{j} + \frac{r}{\sqrt{r^2+p^2}}\vec{k} = \frac{-2p\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}} \qquad \Rightarrow a_n = \frac{-2p\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}} \tag{20}$$

- Acceleration in Cartesian description in coordinate system of helical surface shall be (Eq. 21):

$$\ddot{\vec{r}} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} = a_r\vec{i}_r + a_\theta\vec{i}_\theta + a_n\vec{n} = (\ddot{r} - \dot{r}\dot{\theta}^2)\vec{i}_r + (\sqrt{r^2+p^2}\ddot{\theta} + \frac{2r\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}})\vec{i}_\theta - \frac{2p\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}}\vec{n} \tag{21}$$

At this moment, we can write the forces that act on material point M, in local system of coordinates *xoyz*, on helical surface, as follows:
- Centrifugal force acts on radial direction and has the form (22):

$$\vec{F}_c = m\dot{\theta}^2 r \cdot \vec{i}_r \tag{22}$$

- System gravity force is given by Eq. (23):

$$\vec{G} = -\frac{pmg}{\sqrt{r^2+p^2}}\vec{i}_\theta - \frac{rmg}{\sqrt{r^2+p^2}}\vec{n} \tag{23}$$

- Normal to surface is given by Eq. (24):

$$\vec{N} = N_r\vec{i}_r + N_\theta\vec{i}_\theta + N_n\vec{n} \tag{24}$$

- Friction force is given by Eq. (25):

$$\vec{F}_f = -\mu|N| \cdot \left[\frac{\dot{r}\cdot\vec{i}_r + \sqrt{r^2+p^2}\dot{\theta}\cdot\vec{i}_\theta}{\sqrt{\dot{r}^2 + (r^2+p^2)\dot{\theta}^2}}\right] \tag{25}$$

Movement equations on radial and angular direction are (Eq. 26):

$$\begin{cases} m(\ddot{r} - r\dot{\theta}^2) = m\dot{\theta}^2 r - \mu N \cdot \dfrac{\dot{r}}{\sqrt{\dot{r}^2 + (r^2+p^2)\dot{\theta}^2}} \\ m\sqrt{r^2+p^2}\ddot{\theta} + \dfrac{2mr\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}} = -\dfrac{mgp}{\sqrt{r^2+p^2}} - \mu N \dfrac{\sqrt{r^2+p^2}\dot{\theta}}{\sqrt{\dot{r}^2 + (r^2+p^2)\dot{\theta}^2}} \end{cases} \tag{26}$$

Normal is taken out from equilibrium on the normal direction relation (27):

$$-\frac{2pr\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}}m = -\frac{mgr}{\sqrt{r^2+p^2}} + N \tag{27}$$

From the last equation it can be written the normal expression N and then introduce it in the first two Equations from system (26) in the form (28):

$$N = \frac{mgr}{\sqrt{r^2+p^2}} - \frac{2pr\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}} m \quad (28)$$

Dividing by *m* and i we replace *N* from the last equation in the first two equations and it will result $\ddot{r}$ and $\ddot{\theta}$ (Eqs. 29):

$$\begin{cases} \ddot{r} = 2\dot{\theta}^2 r - \frac{\mu}{\sqrt{r^2+p^2}}(gp - 2pr\dot{r}\dot{\theta})\frac{\dot{r}}{\sqrt{\dot{r}^2+(r^2+p^2)\dot{\theta}^2}} \\ \ddot{\theta} = \frac{-\frac{1}{\sqrt{r^2+p^2}}(gp+\mu gr+2\mu pr\dot{r}\dot{\theta})\cdot\frac{\sqrt{r^2+p^2}\,\dot{\theta}}{\sqrt{\dot{r}^2+(r^2+p^2)\dot{\theta}^2}} - \frac{2r\dot{r}\dot{\theta}}{\sqrt{r^2+p^2}}}{\sqrt{r^2+p^2}} \end{cases} \quad (29)$$

With initial conditions that take into account the speed of material transport to the rotors (v_t), the initial position of material particle (initial radius and angle) r_0, θ_0 (relations (30)), the relations for speed can be written as follows:

$$\begin{aligned} x_0 &= r_0 \cdot \cos(\theta_0) \\ y_0 &= r_0 \cdot \cos(\theta_0) \\ \dot{r}_0 &= vt \cdot \cos(\theta_0) \\ \dot{\theta}_0 &= \omega + \frac{vt \cdot \cos(\theta_0)}{r_0} \end{aligned} \quad (30)$$

In cartesian coordinates, the speed is given by relations (31):

$$\begin{aligned} \dot{x}_0 &= \dot{r}_0 \cos(\theta_0) - r_0 \cdot \dot{\theta}_0 \sin(\theta_0) \\ \dot{y}_0 &= \dot{r}_0 \sin(\theta_0) + r_0 \cdot \dot{\theta}_0 \cos(\theta_0) \end{aligned} \quad (31)$$

where v_t is speed with which each particle reaches the helical surface (conveyor speed).

Eq. system (29) cannot be analytically solved, but only numerically on computer, because it contains the second derivatives of *r* and θ depending on first derivatives and other parameters of the system.

2.3. Motion equations of particle in space

Motion equations in the air are determined by the theory of slanting movement (Cândea et al., 2003) applied on manure spreader apparatus (Eqs. 32):

$$\begin{cases} x(t) = \dot{x}_0(t-t_0) + x_0 \\ y(t) = \dot{y}_0(t-t_0) + y_0 \\ z(t) = -\frac{g}{2}t^2 + (\dot{z}_0 + g\cdot t_0)t + z_0 - \dot{z}_0 t_0 - \frac{g}{2}t_0^{\,2} \end{cases}$$

$$\begin{cases} \dot{x}(t) = \dot{x}_0 \\ \dot{y}(t) = \dot{y}_0 \\ \dot{z}(t) = -g\cdot t + z_0 + gt_0 \end{cases} \quad (32)$$

where *t* is the time passed by a flying particle up to its landing on the soil (h=0, in our case z=0);
t_0 - time after which the particle leaves the rotor from its helical surface described above; $\dot{x}_0$ - initial speed of particle on axis ox; $\dot{y}_0$ - initial speed of particle on axis oy; $\dot{z}_0$ - initial speed of particle on axis oz; z_0 - height from which the particle leaves; *g* - gravitational acceleration.

From the theoretical solving of the third equation from system (32), it results the time after which the particle lands on the soil (z=0).

We shall write down the coefficients of *t* as follows:

$A = \frac{-g}{2}$, $B = \dot{z}_0 + gt_0$ and $C = z_0 - \dot{z}_0 t_0 - \frac{g}{2}t_0^{\,2}$

The resulting solutions are in the form of Eq. (33):

$$\begin{cases} t_1 = \dfrac{-B+\sqrt{B^2+4\cdot A\cdot C}}{2\cdot A} \\ t_2 = \dfrac{-B-\sqrt{B^2+4\cdot A\cdot C}}{2\cdot A} \end{cases} \quad (33)$$

where $t_{landing}$ is maximum between t_1 and t_2

3. Results and discussion

3.1. Working hypotheses

Hypothesis 1) Particle is broken from the whole and remains on helical surface in coordinate points: r_0, y_0 and z_0 or (Eq. 34):

$$\begin{cases} x_0 = r_0 \cos\theta_0 \\ y_0 = r_0 \sin\theta_0 \\ z_0 = \mathrm{K}(r_0) + p\theta_0 \text{ at the moment } t = t_0 \end{cases} \quad (34)$$

r_0 are Θ_0 initial data of particle on helical surface.
It is supposed that from this particular point it starts from resting point: $\dot{x}_0 = \dot{y}_0 = \dot{z}_0 = \dot{r}_0 = \dot{\theta}_0 = 0$

Hypothesis 2) Movement keeps going on helical surface as long as (35):

$$|F_c| < |F_f| \quad (35)$$

(Criterion of leaving the helical surface: centrifugal force has a smaller module than the friction force).

Hypothesis 3) Let us take t_s as moment of time when equality happens (36):

$$|F_c| = |F_f| \quad (36)$$

Then, it is considered that the particle leaves the helical surface under initial conditions (37, 38, 39):

$$\begin{cases} t = t_s \\ r_s = r(t_s) \\ \theta_s = \theta(t_s) \\ \dot{r}_s = \dot{r}(t_s) \\ \dot{\theta}_s = \dot{\theta}(t_s) \end{cases} \quad (37)$$

$$\begin{cases} x_s = r_s \cos\theta_s \\ y_s = r_s \sin\theta_s \\ z_s = p\theta_s \end{cases} \quad (38)$$

$$\begin{cases} \dot{x}_s = \dot{r}_s \cos\theta_s - r_s\dot{\theta}_s \sin\theta_s \\ \dot{y}_s = \dot{r}_s \sin\theta_s + r_s\dot{\theta}_s \cos\theta_s \\ \dot{z}_s = p\dot{\theta}_s \end{cases} \quad (39)$$

Hypothesis 4) Movement after detachment from helical surface takes place according to Eq. (40) (in Cartesian coordinates):

$$m\vec{p} = -\vec{G} \quad (40)$$

where p is helical coil pitch.

In view of the above hypotheses, we shall monitor by means of movement equations on helical surface, the trajectory of particles leaving the disk feeding area. Calculating their trajectories we shall be able to estimate time, coordinates and speed of particle that leave the surface. As criteria of leaving the helical surface by manure particles, at least two may be taken into consideration (but not at the same time):

- particles leaving the surface is given by the condition $r > \phi_1/2$, where ϕ_1 is the coil diameter
- particles leaving the disk surface even for $r < \phi_1/2$, because of aerodynamics force growth over the friction force with surface and gravitational force generated by mass shape and distribution in manure particle.

The second criterion is rather difficult to use, because it requires information on particle aerodynamics characteristics and imposes discussions related to collision phenomena between manure particles, and between them and rotor's surface, when data about collision coefficients between particles and particles and helical surface is compulsory.

Therefore, we suppose that angular speed is equal to particle rotating speed ω in coordinate point given above (Eq. 41):

$$\dot{\theta}_0 = \omega \quad (41)$$

and particle radial speed is expressed by Eq. (42):

$$\dot{r}_0 = \omega r_0 \quad (42)$$

The second criterion is rather difficult to use, because it requires information on particle aerodynamics characteristics and imposes discussions related to collision phenomena between

To solve on the computer by the 4th Runge Kutta numerical method, using the Mathcad program, in which the system of differential Eqs. (26) was introduced in explicit form, i.e. by explaining the second derivatives of r and θ depending on the first derivatives and the other parameters of the system. Eqs. (24) and (32) describe the movement of manure particles from the first contact with the rotor until they reach the soil surface.

Calculations of particle position are complex, so we chose the numerical variant, simulating a small but significant number of particles following their movement on the rotor coil, but also in the air. The calculation procedure achieved in the Mathcad program allows to find the point where the piece will leave the rotor (in polar and cartesian coordinates). Also, il allows the location on the surface of soil on which the piece will land.

During transit of manure particles from the surface of the rotor to the surface of the soil, the speed and direction of the piece are factors that have the greatest influence on the size of the lapse until landing. For the limit conditions, the rotor radius (*r*), the rotational frequency of the rotor (*n*), the rotor distance to the soil (*h*), as well as the coefficient of friction between the manure and the surface it slips on (μ), the angle θ of which the particles leave are written into the program. The result is a distribution model for the organic fertilizer spreaded by the machine. Based on this distribution model, the results can be evaluated and the limit conditions can be changed.

3.2. Components of the movement on helical surface in the local reference system

For exemplification, the following initial data were taken: Rotor speed = 500 rot/min; radius of particle leaving r_0 = 0.115 m; height to the soil h = 1.067m (for θ=110°); the inclination of the rotor relative to the vertical axis - 10°.

- Radial coordinate:

Fig. 4 shows the variation in time of the radial coordinate on the disk in the time interval between the initial moment and the moment of detachment from the disk, for the dimensions of the machine MG 5, r_0 = 0.015m and t_e = 0.0138 s.

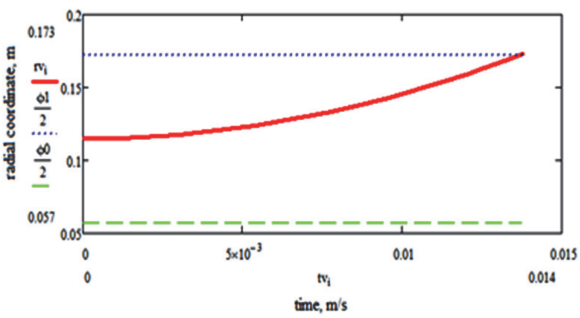


Fig. 4. Variation in time of the radial coordinate on the helical surface

- Angular coordinate:

Angular coordinate θ, increases in the mode from -110 ° to -150 ° ie by 40 ° from time t_0 = 0 to t_e = 0.014 s (the minus sign is due to the negative sense considered for calculation) as shown in Fig. 5.

- Radial speed

Fig. 6 presents the variation in time of the radial speed of the particle on the coil in the time interval between the initial moment and the moment of detachment, reaching 0.014 s from 0 to 8.428 m/s. Fig. 7 shows the trajectory of a fertilizer particle on the helical surface of rotor IV (in Fig. 1), whose coordinates are given in Fig. 5 and Fig. 6.

3.3. Components of motion in air

The two movements being studied separately we consider that particle position and speed at the end of motion on helical surface is the initial conditions of motion for the second path (air). Fig. 8 (a), (b) and (c) show the initial coordinates of particle motion in the air, landing coordinates and landing time.

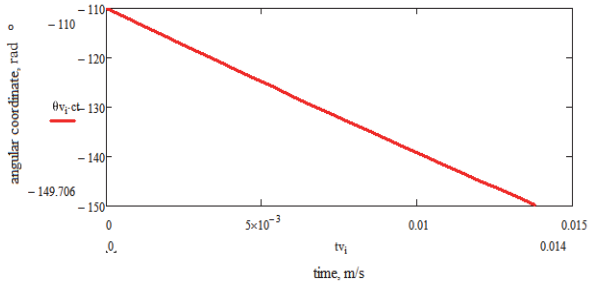


Fig. 5. Variation in time of the angular coordinate on the helical surface

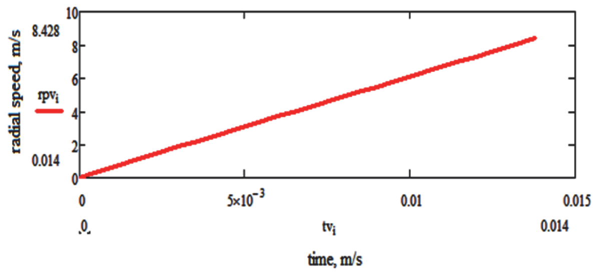


Fig. 6. Variation in time of the radial speed on the helical surface

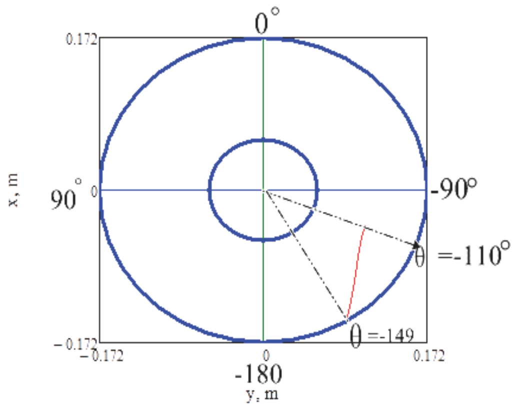


Fig. 7. Trajectory of a fertilizer particle on the helical surface

The final trajectory composed of the two trajectories, on the disk and in the air, has the following shape illustrated in Fig. 9. The coordinates where the point reaches the soil are: x = 5.178 m, y = 1.82 m. So the rotor IV throws the particle at a distance of about 5 m behind the machine and 1.82 m on the left of the machine, from the first coil.

4. Conclusions

By means of the proposed mathematical model it could be possible to analyze the particle distribution on soil by performing a series of numerical simulations.

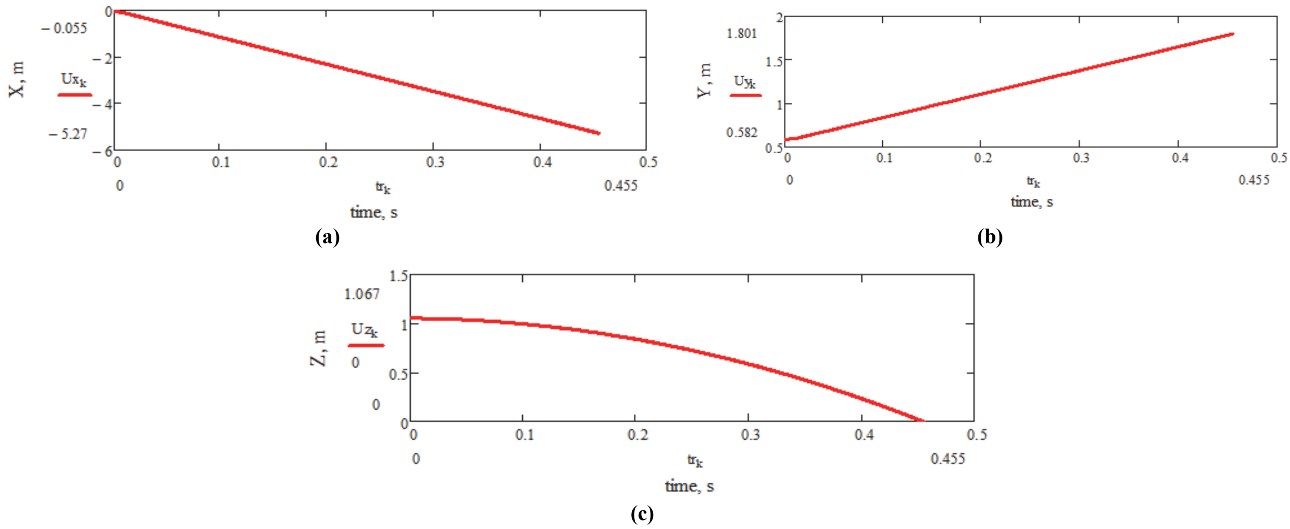


Fig. 8. Coordinates x, y şi z of movement in space: (a) Coordinate on OX axis: The particle leaves from x=0.055 m and reaches to 5.27 m when landing on the soil; (b) Coordinate on OY axis: The particle leaves from y=0.582 m and reaches to 1.8 m when landing on the soil; (c) Coordinate on OZ axis: The particle leaves from z=1,067 m and reaches to 0 m, when it reaches the soil

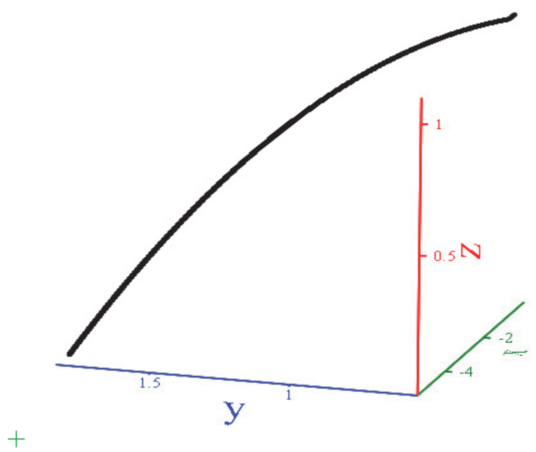


Fig. 9. The trajectory of the particle from when it leaves the rotor until landing on the soil

It can be shown the interdependence between the size of distribution apparatus (spiral diameter, pitch, length, inclination angle and distance between rotors), material starting position, disk rotation frequency, material friction coefficient and distance from the soil to the disk. The comparison between the results of the mathematical model and the real distribution can be affected by a series of variables: the material homogeneity, the maintaining of a constant flow of material, maintaining the same rotational speed of the rotors and the same tractor working speed during the distribution. The collisions between particles were also not considered but it is expected to influence the distribution. These aspects will be subject to the following research.

Mathematical model can be improved over the time, sometimes initial conditions having errors because the process of taking over the manure fertiliser particles by the disk has a random character.

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